Mathematics

Question 1

With usual notations, in any $\triangle ABC$, if $a \cos B = b \cos A$, then the triangle is

Options:

A. an isosceles triangle

B. an equilateral triangle

C. a right angled triangle

D. a scalene triangle

Answer: A

Solution:

Solution:
Using sine rule
$\frac{a}{\sin A} = \frac{b}{\sin B} = K$
We have, $\frac{\cos A}{a} = \frac{\cos B}{b}$
$\Rightarrow \frac{\cos A}{K \sin A} = \frac{\cos B}{K \sin B}$
$\Rightarrow K \sin B \cos A = K \sin A \cos B$
$\Rightarrow \cos A \sin B - \cos B \sin A = 0$
$\Rightarrow \sin(A - B) = 0$
$A = B$
$\therefore$ Triangle in an isosceles triangle.

Question 2

If θ lies in the first quadrant and $5 \tan \theta = 4$, then $\frac{5 \sin \theta - 3 \cos \theta}{\sin \theta + 2 \cos \theta}$ is equal to

Options:

A. $\frac{5}{14}$

B. $\frac{3}{14}$

C. $\frac{1}{14}$

D. 0

Answer: A

Solution:

Solution:

Given, $\tan\theta = \frac{4}{5}$

$\therefore \sin\theta = \frac{4}{\sqrt{41}}$ and $\cos\theta = \frac{5}{\sqrt{41}}$

Now, $\frac{5\sin\theta - 3\cos\theta}{\sin\theta + 2\cos\theta} = \frac{5 \times \frac{4}{\sqrt{41}} - 3 \times \frac{5}{\sqrt{41}}}{\frac{4}{\sqrt{41}} + 2 \times \frac{5}{\sqrt{41}}} = \frac{5}{14}$

Question 3

If $\cos^{-1}x > \sin^{-1}x$, then

Options:

A. $\frac{1}{\sqrt{2}} < x \leq 1$

B. $0 \leq x \leq \frac{1}{\sqrt{2}}$

C. $-1 \leq x < \frac{1}{\sqrt{2}}$

D. $x > 0$

Answer: C

Solution:

Solution:

$\cos^{-1}x > \sin^{-1}x$[where $x \in [-1, 1]$]

$\because \cos^{-1}x = \frac{\pi}{2} - \sin^{-1}x$

$\therefore \frac{\pi}{2} - \sin^{-1}x > \sin^{-1}x \Rightarrow 2\sin^{-1}x < \frac{\pi}{2}$

$\Rightarrow \sin^{-1}x < \frac{\pi}{4} \Rightarrow x < \frac{1}{\sqrt{2}}$

$\because -1 \leq x \leq 1$

$-1 \leq x < \frac{1}{\sqrt{2}}$

Question 4

The value of $\tan 3A - \tan 2A - \tan A$ is

Options:

A. $\tan 3A \tan 2A \tan A$

B. $-\tan 3A \tan 2A \tan A$

C. $\tan A \tan 2A - \tan 2A \tan 3A$

D. $\tan 3A \tan A - \tan 2A \tan 3A$

Answer: A

Solution:

Solution:
Let $3A = A + 2A$
$\tan 3A = \tan(A + 2A)$
$\tan 3A = \dfrac{\tan A + \tan 2A}{1 - \tan A \cdot \tan 2A}$
$\Rightarrow \tan A + \tan 2A = \tan 3A - \tan 3A \tan 2A \cdot \tan A$
$\Rightarrow \tan 3A - \tan 2A - \tan A = \tan 3A \cdot \tan 2A \cdot \tan A$

Question 5

The three straight lines $ax + by = c$, $bx + cy = a$ and $cx + ay = b$ are collinear, if

Options:

A. $b + c = a$

B. $c + a = b$

C. $a + b + c = 0$

D. $a + b = c$

Answer: C

Solution:

Solution:
Given lines are $ax + by = c$, $bx + cy = a$ and $cx + ay = b$
On adding the given three equations, we get
$ax + by + bx + cy + cx + ay = a + b + c$
$\Rightarrow (a + b + c)x + (a + b + c)y = (a + b + c)$
On comparing with $0x + 0y = 0$ for collinearity, we get
$a + b + c = 0$

Question 6

A line through the point $A(2, 0)$ which makes an angle of 30° with the positive direction of X-axis is rotated about A in clockwise direction

through an angle of 15°. Then, the equation of the straight line in the new position is

Options:

A. $(2-\sqrt{3})x+y-4+2\sqrt{3}=0$

B. $(2-\sqrt{3})x-y-4+2\sqrt{3}=0$

C. $(2-\sqrt{3})x-y+4+2\sqrt{3}=0$

D. $(2-\sqrt{3})x+y+4+2\sqrt{3}=0$

Answer: B

Solution:

Solution:
The equation of line in new position is
$y-0=\tan 15^\circ(x-2)$
$\Rightarrow y=(2-\sqrt{3})(x-2)$
$\Rightarrow (2-\sqrt{3})x-y-4+2\sqrt{3}=0$

Question 7

Find the value of $\cos\left(\frac{x}{2}\right)$, if $\tan x=\frac{5}{12}$ and x lies in third quadrant.

Options:

A. $\frac{5}{\sqrt{13}}$

B. $\frac{5}{\sqrt{26}}$

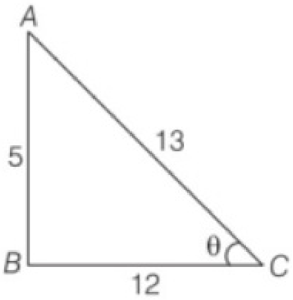
C. $\frac{5}{13}$

D. $\sqrt{\frac{1}{26}}$

Answer: D

Solution:

Solution:
Given, $\tan x=\frac{5}{12}$ and x lies in III quadrant.
$\therefore \sin x=\frac{-5}{13}$ and $\cos x=\frac{-12}{13}$



Now, $\cos x = 2\cos^2 \frac{x}{2} - 1$

$\Rightarrow \cos^2 \frac{x}{2} = \frac{1}{2}(\cos x + 1)$

$= \frac{1}{2}\left(\frac{-12}{13} + 1\right) = \frac{1}{2}\left(\frac{1}{13}\right) = \frac{1}{26}$

$\therefore \cos \frac{x}{2} = \sqrt{\frac{1}{26}}$

Question 8

The locus of a point which moves, so that the ratio of the length of the tangents to the circles $x^2 + y^2 + 4x + 3 = 0$ and $x^2 + y^2 - 6x + 5 = 0$ is 2 : 3, is

Options:

A. $5x^2 + 5y^2 - 60x + 7 = 0$

B. $5x^2 + 5y^2 + 60x - 7 = 0$

C. $5x^2 + 5y^2 - 60x - 7 = 0$

D. $5x^2 + 5y^2 + 60x + 7 = 0$

Answer: D

Solution:

Solution:

Since, $\frac{\sqrt{S_1}}{\sqrt{S_2}} = \frac{2}{3}$

$\therefore \frac{\sqrt{x_1^2 + y_1^2 + 4x_1 + 3}}{\sqrt{x_1^2 + y_1^2 - 6x_1 + 5}} = \frac{2}{3}$

$\Rightarrow 9x_1^2 + 9y_1^2 + 36x_1 + 27 - 4x_1^2 - 4y_1^2 + 24x_1 - 20 = 0$

$\Rightarrow 5x_1^2 + 5y_1^2 + 60x_1 + 7 = 0$

$\therefore$ Locus of point (x, y) is

$5x^2 + 5y^2 + 60x + 7 = 0$

Question 9

The circles $x^2 + y^2 + 6x + 6y = 0$ and $x^2 + y^2 - 12x - 12y = 0$

Options:

A. cut orthogonally

B. touch each other internally

C. intersect two points

D. touch each other externally

Answer: D

Solution:

Solution:
The centres of given circles are $C_1(-3, -3)$, $C_2(6, 6)$ and radii are
$r_1 = \sqrt{9 + 9 + 0} = 3\sqrt{2}$, $r_2 = \sqrt{36 + 36 + 0} = 6\sqrt{2}$, respectively.
Now, $C_1C_2 = \sqrt{(6 + 3)^2 + (6 + 3)^2} = 9\sqrt{2}$
and $r_1 + r_2 = 3\sqrt{2} + 6\sqrt{2} = 9\sqrt{2}$
Here, $C_1C_2 = r_1 + r_2$
So, both circles touch each other externally.

Question 10

The mean and variance of n observations $x_1, x_2, x_3, \ldots, x_n$ are 5 and 0 , respectively. If $\sum_{x=1}^{n} x_i^2 = 400$, then the value of n is equal to

Options:

A. 80

B. 25

C. 20

D. 16

Answer: D

Solution:

Solution:
$\bar{x} = 5$, variance $= \frac{1}{n}\Sigma x_i^2 - (\bar{x})^2$
$\Rightarrow \quad 0 = \frac{1}{n} \cdot 400 - 25$
$\Rightarrow n = \frac{400}{25} = 16$

Question 11

The variance of first n natural numbers is

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Options:

A. $\frac{n(n+1)}{2}$

B. $\frac{(n+1)(n+5)}{12}$

C. $\frac{(n+1)(n-5)}{12}$

D. $\frac{(n^2-1)}{12}$

Answer: D

Solution:

Solution:
Variance of first n natural numbers

$$= \frac{\Sigma n^2}{n} - \left(\frac{\Sigma n}{n}\right)^2$$

$$= \frac{n(n+1)(2n+1)}{6n} - \left(\frac{n(n+1)}{2n}\right)^2$$

$$= (n+1)\left[\frac{2n+1}{6} - \frac{(n+1)}{4}\right]$$

$$= \frac{(n+1)}{12}[4n+2-3n-3]$$

$$= \frac{(n+1)}{12} \times (n-1) = \frac{n^2-1}{12}$$

Question 12

Out of 50 tickets numbered 00, 01, 02, ..., 49, one ticket is drawn randomly, the probability of the ticket having the product of its digits 7 , given that the sum of the digits is 8 , is

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Options:

A. $\frac{1}{14}$

B. $\frac{3}{14}$

C. $\frac{1}{5}$

D. None of these

Answer: C

Solution:

Solution:
Total number of cases $= {}^{50}C_1 = 50$
Let A be the event of selecting ticket with sum of digits ' 8 '.
Favourable cases to A are $\{08, 17, 26, 35, 44\}$.
Let B be the event of selecting ticket with product of its digits ' 7 '.
Favourable cases to B is only $\{17\}$.
Now, $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{1/50}{5/50} = \frac{1}{5}$

Question 13

The probability of India winning a test match against South Africa is 1/2 assuming independence form match to match played. The probability that in a match series India's second win occurs at the third day is

Options:

A. $\frac{1}{8}$

B. $\frac{1}{2}$

C. $\frac{1}{4}$

D. $\frac{2}{3}$

Answer: C

Solution:

Solution:
Given, probability of winning a test match, $P(W) = \frac{1}{2}$
Probability of lossing a match, $P(L) = 1 - \frac{1}{2} = \frac{1}{2}$
Probability that India's second win occurs at the third day
$= P(L) \cdot P(W) \cdot P(W) + P(W) \cdot P(L) \cdot P(W)$
$= \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8} + \frac{1}{8} = \frac{1}{4}$

Question 14

Consider the following statements
r : If a number is a multiple of 9 , then it is multiple of 3.
Let p and q denote the statements
p: A number is a multiple of 9.
q : A number is a multiple of 3 .
Then, if p then q is the same as

Options:

A. p only if q

B. q is a necessary condition for p

C. ~q implies ~p

D. All of the above

Answer: D

Solution:

Solution:
I. p only if q. This says that number is a multiple of 9 only, if it is a multiple of 3.
II. q is a necessary condition for p.
This says that when a number is a multiple of 9 , it is necessarily a multiple of 3 .
III. ~q implies ~p. This says that if a number is not a multiple of 3 , then it is not a multiple of 9 .

Question 15

The negation of the statement 7 is greater than 4 or 6 is less than 7 .

Options:

A. 7 is not greater than 4 and 6 is not less than 7

B. 7 is not greater than 4 or 6 is not less than 7

C. 7 is greater than 4 and 6 is less than 7

D. None of the above

Answer: A

Solution:

Solution:
Let p : 7 is greater than 4
and q : 6 is less than 7 .
Then, the given statement is disjunction $p \vee q$.
Here, ~p : 7 is not greater than 4 .
and ~q : 6 is not less than 7 .
$\therefore \sim(p \vee q)$: 7 is not greater than 4 and 6 is not less than 7 .

Question 16

For an invertible matrix A, if A(ad jA) = $\left|\begin{matrix} 20 & 0 \\ 0 & 20 \end{matrix}\right|$**, then |A| =**

Options:

A. 20

B. −200

C. 200

D. −20

Answer: A

Solution:

Solution:

Given, $A(\operatorname{ad} jA) = \begin{vmatrix} 20 & 0 \\ 0 & 20 \end{vmatrix}$

$20 \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 20/$

We know that

$A^{-1} = \frac{1}{|A|} \operatorname{ad} jA$

$\Rightarrow A(\operatorname{ad} jA) = |A| I$

$\Rightarrow 20I = |A| I$

$\Rightarrow |A| = 20$

Question 17

If matrix $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$, then $A^2 - 4A + 5I$ is where I is a unit matrix.

Options:

A. Null matrix

B. Skew symmetric matrix

C. symmetric Matrix

D. None of the above

Answer: A

Solution:

Solution:

Given, $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

$\therefore A^2 = A \cdot A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

$= \begin{bmatrix} 1 \times 1 + (-1) \times 2 & 1 \times (-1) + (-1) \times 3 \\ 2 \times 1 + 3 \times 2 & 2 \times (-1) + 3 \times 3 \end{bmatrix}$

$$= \begin{bmatrix} 1-2 & -1-3 \\ 2+6 & -2+9 \end{bmatrix} = \begin{bmatrix} -1 & -4 \\ 8 & 7 \end{bmatrix}$$

Now, $A^2 - 4A + 5I$

$$= \begin{bmatrix} -1 & -4 \\ 8 & 7 \end{bmatrix} - 4\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} + 5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -4 \\ 8 & 7 \end{bmatrix} - \begin{bmatrix} 4 & -4 \\ 8 & 12 \end{bmatrix} + \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} -1-4+5 & -4+4+0 \\ 8-8+0 & 7-12+5 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

Question 18

If $f(x) = \begin{cases} \frac{3\sin \pi x}{5x} & x \neq 0 \\ 2k & x = 0. \end{cases}$ is continuous at $x = 0$, then the value of k is

Options:

A. $\frac{\pi}{10}$

B. $\frac{3\pi}{10}$

C. $\frac{3\pi}{2}$

D. $\frac{3\pi}{5}$

Answer: D

Solution:

Solution:

Given, $f(x) = \begin{cases} \frac{3\sin \pi x}{5x} & x \neq 0 \\ 2k & x = 0. \end{cases}$

Now, $\lim\limits_{x \to 0} f(x) = \lim\limits_{x \to 0} \left(\frac{3\sin \pi x}{5x} \right)$

$= \frac{3}{5}\lim\limits_{x \to 0} \left(\sin \frac{\pi x}{\pi x} \right) \times \pi = \frac{3}{5} \times 1 \times \pi = \frac{3}{5}\pi$

Also, $f(0) = 2k$

Since, $f(x)$ is continuous at $x = 0$.

$\therefore f(0) = \lim\limits_{x \to 0} f(x)$

$\Rightarrow 2k = \frac{3}{5}\pi \Rightarrow k = \frac{3\pi}{10}$

Question 19

If f (x) = $\begin{cases} \frac{\sin^3(\sqrt{3}) \cdot \log(1+3x)}{(\tan^{-1}\sqrt{x})^2 \left(e^{5\sqrt{3}}-1\right)x} & x \neq 0 \\ a & x = 0. \end{cases}$ **is continuous in [0, 1] then a equals**

Options:

A. 0

B. $\frac{3}{5}$

C. 2

D. $\frac{5}{3}$

Answer: B

Solution:

Solution:

We have, $f(x) = \begin{cases} \frac{\sin^3(\sqrt{3}) \cdot \log(1+3x)}{(\tan^{-1}\sqrt{x})^2 \left(e^{5\sqrt{3}}-1\right)x} & x \neq 0 \\ a & x = 0. \end{cases}$

For continuity in [0, 1], $f(0) = \lim_{x \to 0} f(x)$ otherwise it is discontinuous.

$\therefore\ a \ = \lim_{x \to 0} \frac{\sin^3(\sqrt{x}) \cdot \log(1+3x)}{x(\tan^{-1}\sqrt{x})^2 \cdot \left(e^{5\sqrt{x}}-1\right)}$

$= \lim_{x \to 0} \left[\frac{3}{5} \cdot \frac{\sin^3\sqrt{x}}{(\sqrt{x})^3} \cdot \frac{(\sqrt{x})^3}{(\tan^{-1}\sqrt{x})^3} \right.$

$\& \times \left. \frac{\log(1+3x)}{3x} \cdot \frac{5\sqrt{x}}{3^{5\sqrt{x}}-1} \right]$

$= \frac{3}{5} \lim_{x \to 0} \frac{\sin^3}{(\sqrt{x})^3} \cdot \frac{(\sqrt{x})^3}{\tan^{-1}\sqrt{x}}$

$\therefore\ a \ = \frac{3}{5}$

Question 20

The differential equation of all parabolas having vertex at the origin and axis along positive Y -axis is

Options:

A. $x^2 \frac{dy}{dx} - y = 0$

B. $x \frac{dy}{dx} + 2y = 0$

C. $x \frac{dy}{dx} = 2y$

D. $x^3 \frac{dy}{dx} = 3y$

Answer: C

Solution:

Solution:
The general equation of an parabola having vertex at the origin and axis along positive Y -axis is
$x^2 = 4ay. . . (i)$
On differentiating Eq. (i), we get
$2x = 4a \frac{dy}{dx}$
$\Rightarrow \frac{dy}{dx} = \frac{x}{2a} \Rightarrow 2a = \frac{x}{dy/dx}$
Putting value of 2a in Eq. (i), we get
$x^2 = 2\left(\frac{x}{dy/dx}\right)y \Rightarrow x\frac{dy}{dx} = 2y$

Question 21

The order and degree of the differential equation $\sqrt{\frac{dy}{dx}} - 4\frac{dy}{dx} - 7x = 0$ are

Options:

A. 1 and 1/2

B. 2 and 1

C. 1 and 1

D. 1 and 2

Answer: D

Solution:

Solution:
Given equation is $\sqrt{\frac{dy}{dx}} - 4\frac{dy}{dx} - 7x = 0$
$\Rightarrow \frac{dy}{dx} = 16\left(\frac{dy}{dx}\right)^2 + 49x^2 + 56x\frac{dy}{dx}$
Obviously, it is of first order and second degree differential equation.

Question 22

The solution of the differential equation $e^{-x}(y+1)dy + (\cos^2 x - \sin 2x)y\,dx = 0$ subjected to the condition that $y = 1$, when $x = 0$ is

Options:

A. $y + \log y + e^x\cos^2 x = 2$

B. $\log(y+1) + e^x\cos^2 x = 1$

C. $y + \log y = e^x\cos^2 x$

D. $(y+1) + e^x\cos^2 x = 2$

Answer: A

Solution:

Solution:
Given equation can be rewritten as
$\left(1 + \frac{1}{y}\right)dy = -e^x(\cos^2 x - \sin 2x)dx$
On integrating both sides, we get
$y + \log y = -e^x\cos^2 x + \int e^x \sin 2x\, dx$
$-\int e^x \sin 2x\, dx + C$
$\Rightarrow\ y + \log y = -e^x\cos^2 x + C$
At $x = 0$ and $y = 1$,
$1 + 0 = -e^0 \cos 0 + C$
$C = 2$ [given]
∴ Required solutions is
$y + \log y = -e^x\cos^2 x + 2$
$\Rightarrow\ y + \log y + e^x\cos^2 x = 2$

Question 23

$\int_{-1}^{1} \frac{17x^5 - x^4 + 29x^3 - 31x + 1}{x^2+1}dx$ **is equal to**

Options:

A. $\frac{4}{5}$

B. $\frac{5}{4}$

C. $\frac{4}{3}$

D. $\frac{3}{4}$

Answer: C

Solution:

Solution:
$\int_{-1}^{1} \frac{17x^5 - x^4 + 29x^3 - 31x + 1}{x^2+1}dx$

$$= \int_{-1}^{1} \underbrace{\frac{17x^5 + 29x^3 - 31x}{x^2 + 1}}_{\text{Odd function}} dx - \int_{-1}^{1} \underbrace{\frac{x^4 - 1}{x^2 + 1}}_{\text{Even function}} dx$$

$$= 0 - 2\int_0^1 \frac{(x^2 - 1)(x^2 + 1)}{(x^2 + 1)} dx = -2\left[\left(\frac{x^3}{3} - x\right)\right]_0^1 = \frac{4}{3}$$

Question 24

The number of ways of arranging letters of the 'HAVANA', so that V and N do not appear together, is

Options:

A. 60

B. 80

C. 100

D. 120

Answer: B

Solution:

Solution:
Given word is 'HAVANA' (3A, 1H, 1N, 1V)
Total number of ways of arranging the given word
$= \frac{6!}{3!} = 120$
Total number of words in which N, V together
$= \frac{5!}{3!} \times 2! = 40$
∴ Required number of ways $= 120 - 40 = 80$

Question 25

Out of 7 consonants and 4 vowels, the number of words (not necessarily meaningful) that can be made, each consisting of 3 consonants and 2 vowels, is

Options:

A. 24800

B. 25100

C. 25200

D. 25400

Answer: C

Solution:

Solution:
3 consonants can be selected from 7 consonants $= {}^7C_3$ ways
2 vowels can be selected from 4 vowels $= {}^4C_2$ ways
∴ Required number of words $= {}^7C_3 \times {}^4C_2 \times 5\,!$
[selected 5 letters can be arranged in 5 ! ways, to get a different word]
$= 35 \times 6 \times 120 = 25200$

Question 26

If $f(x) = \sin^2 x + \sin^2\left(x + \frac{\pi}{3}\right) + \cos x \cos\left(x + \frac{\pi}{3}\right)$ and $g\left(\frac{5}{4}\right) = 1$, then $gof(x)$ is equal to

Options:

A. 1

B. −1

C. 2

D. −2

Answer: A

Solution:

Solution:
Given $f(x) = \sin^2 x + \sin^2\left(x + \frac{\pi}{3}\right) + \cos x$
$\cos\left(x + \frac{\pi}{3}\right)$
$= \sin^2 x + \left[\sin x \cos \frac{\pi}{3} + \cos x \sin \frac{\pi}{3}\right]^3$
$+ \cos x\left[\cos x \cos \frac{\pi}{3} - \sin x \sin \frac{\pi}{3}\right]$
$= \sin^2 x + \left[\frac{\sin x}{2} + \cos x \cdot \frac{\sqrt{3}}{2}\right]^2$
∴ $= \cos x\left[\frac{\cos x}{2} - \sin x \cdot \frac{\sqrt{3}}{2}\right]$
$= \sin^2 x + \frac{\sin^2 x}{4} + \frac{3\cos 2}{4} + \sin x \cos x \cdot \frac{\sqrt{3}}{2}$
$= \frac{5\sin^2 x}{4} + \frac{5\cos^2 x}{4} = \frac{5}{4}$
$\Rightarrow (x) = g[f(x)] = g\left(\frac{5}{4}\right) = 1$

Question 27

Range of the function $f(x) = \frac{x}{1 + x^2}$ is

Options:

A. $(-\infty, \infty)$

B. $[-1, 1]$

C. $\left[-\frac{1}{2}, \frac{1}{2}\right]$

D. $[-\sqrt{2}, \sqrt{2}]$

Answer: C

Solution:

Solution:

Let $y = \frac{x}{1+x^2} \Rightarrow x^2y - x + y = 0$

For x to be real, $1 - 4y^2 \geq 0$ { Discriminant $= 1 - 4y^2$ }

$\Rightarrow 4y^2 - 1 \leq 0$

$\Rightarrow (2y-1)(2y+1) \leq 0$

$\Rightarrow -\frac{1}{2} \leq y \leq \frac{1}{2}$

$\therefore y = f(x) \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

Question 28

The domain of $f(x) = \sin^{-1}\left[\log_2\left(\frac{x}{2}\right)\right]$ is

Options:

A. $0 \leq x \leq 1$

B. $0 \leq x \leq 4$

C. $1 \leq x \leq 4$

D. $4 \leq x \leq 6$

Answer: C

Solution:

Solution:

Given, $f(x) = \sin^{-1}\left[\log_2\left(\frac{x}{2}\right)\right]$

$\Rightarrow -1 \leq \log_2\left(\frac{x}{2}\right) \leq 1 \Rightarrow 2^{-1} \leq \frac{x}{2} \leq 2^1$

$\Rightarrow \frac{1}{2} \leq \frac{x}{2} \leq 2 \Rightarrow 1 \leq x \leq 4$

Question 29

If $f(x) = \frac{k}{2^x}$ is a probability distribution of a random variable X that can take on the values x = 0, 1, 2, 3, 4. Then, k is equal to

Options:

A. $\frac{16}{15}$

B. $\frac{15}{16}$

C. $\frac{31}{16}$

D. None of these

Answer: D

Solution:

Solution:

We have, $f(x) = \frac{k}{2^x}, x = 0, 1, 2, 3, 4$

Since, f (x) is a probability distribution of a random variable X , therefore we have

$\sum_{x=0}^{4} f(x) = 1 \Rightarrow \sum_{x=0}^{4} \left(\frac{k}{2^x} \right) = 1$

$\Rightarrow k \sum_{x=0}^{4} \frac{1}{2^x} = 1$

$\Rightarrow k\left(1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4}\right) = 1$

$\Rightarrow k\left(\frac{16 + 8 + 4 + 2 + 1}{2^4}\right) = 1$

$\Rightarrow k \times \left(\frac{31}{16}\right) = 1$

$\therefore k = \frac{16}{31}$

Question 30

$\lim_{x \to 0} \frac{e^{x^2} - \cos x}{x^2}$ **is equal to**

Options:

A. 0

B. $\frac{1}{2}$

C. 1

D. $\frac{3}{2}$

Answer: D

Solution:

Solution:

$$\lim_{x \to 0} \frac{e^{x^2} - \cos x}{x^2} \left[\frac{0}{0} \text{ form } \right] \text{ [using L'Hospital's rule]}$$

$$= \lim_{x \to 0} \frac{2xe^{x^2} + \sin x}{2x} \left[\frac{0}{0} \text{ form } \right]$$

$$= \lim_{x \to 0} \frac{2e^{x^2} + 4x^2 e^{x^2} + \cos x}{2}$$

$$= \frac{2 + 0 + 1}{2} = \frac{3}{2} \text{ [using L'Hospital's rule]}$$

Question 31

Evaluate $\lim_{x \to \sqrt{2}} \frac{x^4 - 4}{x^2 + 3\sqrt{2}x - 8}$.

Options:

A. $\frac{8}{5}$

B. $\frac{8}{3}$

C. $\frac{5}{8}$

D. $\frac{3}{8}$

Answer: A

Solution:

Solution:

$$\lim_{x \to \sqrt{2}} \frac{x^4 - 4}{x^2 + 3\sqrt{2}x - 8}$$

$$= \lim_{x \to \sqrt{2}} \frac{(x + \sqrt{2})(x - \sqrt{2})(x^2 + 2)}{(x - \sqrt{2})(x + 4\sqrt{2})}$$

$$= \lim_{x \to \sqrt{2}} \frac{(x + \sqrt{2})(x^2 + 2)}{(x + 4\sqrt{2})}$$

$$= \frac{(\sqrt{2} + \sqrt{2})(2 + 2)}{(\sqrt{2} + 4\sqrt{2})} = \frac{(2\sqrt{2})(4)}{5\sqrt{2}} = \frac{8\sqrt{2}}{5\sqrt{2}} = \frac{8}{5}$$

Question 32

What will be projection of the vector $4\hat{i} - 3\hat{j} + \hat{k}$ on the line joining the

points (2, 3, −1) and (−2, −4, 3) ?

Options:

A. 1

B. 2

C. 3

D. 4

Answer: A

Solution:

Solution:
Let point A = (2, 3, −1) and point B = (−2, −4, 3).
Now the position vector of line joining A and B,
$AB = OB - OA$
$= -2\hat{i} - 4\hat{j} + 3\hat{k} - 2\hat{i} - 3\hat{j} + \hat{k}$
$= -4\hat{i} - 7\hat{j} + 4\hat{k}$
Again let $a = AB = -4\hat{i} - 7\hat{j} + 4\hat{k}$
and $b = 4\hat{i} - 3\hat{j} + \hat{k}$
Then, $a \cdot b = \left(-4\hat{i} - 7\hat{j} + 4\hat{k}\right) \cdot \left(4\hat{i} - 3\hat{j} + \hat{k}\right)$
$= -16 + 21 + 4 = 9$
$|a| = \sqrt{(-4)^2 + (-7)^2 + (4)^2}$
$= \sqrt{16 + 49 + 16} = 9$
Now projection of b on a
$= \frac{(a \cdot b)}{|a|} = \frac{9}{9} = 1$

Question 33

If $a = 3\hat{i} - 2\hat{j} + \hat{k}$ and $b = 2\hat{i} - 4\hat{j} - 3\hat{k}$, then $|a - 2b|$ will be

Options:

A. 9

B. $\sqrt{86}$

C. $\sqrt{94}$

D. 10

Answer: B

Solution:

Solution:

$a = 3\hat{i} - 2\hat{j} + \hat{k}$ . . . (i)
$b = 2\hat{i} - 4\hat{j} - 3\hat{k}$ . . . (ii)
Multiplying by 2 both sides, we get
$2b = 4\hat{i} - 8\hat{j} - 6\hat{k}$ . . . (iii)
Subtracting Eq. (iii) from Eq. (i), we get
$a - 2b = \left(3\hat{i} - 2\hat{j} + \hat{k}\right) - \left(4\hat{i} - 8\hat{j} - 6\hat{k}\right)$
$= -\hat{i} + 6\hat{j} + 7\hat{k}$
$= \sqrt{(-1)^2 + (6)^2 + (7)^2} = \sqrt{86}$

Question 34

What will be the equation of plane passing through a point (1, 4, −2) and parallel to the given plane $-2x + y - 3z = 9$?

Options:

A. $2x - y + 3z + 8 = 0$

B. $2x = y + 3z + 8 = 0$

C. $2x - 2y + 2z + 10 = 0$

D. $2x + 2y - 3z + 8 = 0$

Answer: A

Solution:

Solution:
$r \cdot \left(2\hat{i} - 3\hat{j} + 5\hat{k}\right) + 2 = 0$
Equation of plane parallel to the given plane is
$-2x + y - 3z = \lambda$. . . (i)
Since, plane (i) passes through the point (1, 4, −2).
Therefore this point satisfy the given plane. Put $x = 1$, $y - 4$ and $z = -2$ in Eq. (i)
$-2(1) + 4 - 3(-2) = \lambda$
$-2 + 4 + 6 = \lambda$
$\lambda = 8$
Put $\lambda = 8$ in Eq. (i), we get
$-2x + y - 3z = 8$
$2x - y + 3z + 8 = 0$

Question 35

If the line joining two points A(2, 0) and B(3, 1) is rotated about A in anticlockwise direction through an angle of 15°, then the equation of the line in new position is

Options:

A. $y = -\sqrt{3}x + 2\sqrt{3}$

B. $y = 3x - 6$

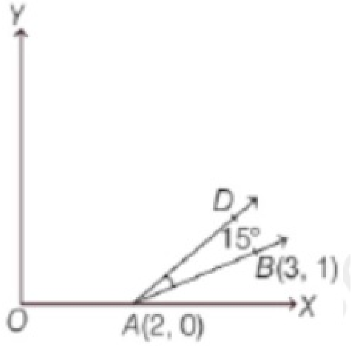
C. $y = \frac{1}{\sqrt{3}}x - \frac{2}{\sqrt{3}}$

D. $y = \sqrt{3}x - 2\sqrt{3}$

Answer: D

Solution:

Solution:



The slope of the line is $\frac{y_2 - y_1}{x_2 - x_1} = \frac{1-0}{3-2} = 1$ or $\tan 45^\circ$ rotated anti-clockwise direction the line through 15° the slope of line AD in new position. New position will be $\tan 60^\circ = \sqrt{3}$.
Therefore, the equation of the new line AD is
$y - 0 = \sqrt{3}(x - 2)$
$y = \sqrt{3}x - 2\sqrt{3}$

Question 36

$\int \frac{(\log x)^2}{x} \mathbf{d\,x}$

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Options:

A. $\left(\frac{\log x}{x}\right)^3 + C$

B. $\left(\frac{\log x}{3}\right)^3 + C$

C. $\frac{(\log x)^3}{2} + C$

D. $\frac{(\log x)^3}{3} + C$

Answer: D

Solution:

Solution:
Let $I = \int \frac{(\log x)^2}{x} d\,x$
Put $\log x = t \Rightarrow \frac{1}{x} d\,x = d\,t$

$\therefore\ I = \int \frac{(\log x)^2}{x} dx = \int t^2 dt$

$= \frac{t^3}{3} + C = \frac{(\log x)^3}{3} + C \quad [\text{put } t = \log x]$

Question 37

$\int \frac{\tan^4\sqrt{x} \cdot \sec^2\sqrt{x}}{\sqrt{x}} dx =$

©

Options:

A. $\frac{-5}{2}[\tan\sqrt{x}]^5 + C$

B. $[\tan\sqrt{x}]^5 + C$

C. $\frac{2}{5}[\tan\sqrt{x}]^5 + C$

D. $\frac{5}{2}[\tan\sqrt{x}]^5 + C$

Answer: C

Solution:

Solution:

Let $I = \int \frac{\tan^4\sqrt{x} \cdot \sec^2\sqrt{x}}{\sqrt{x}} dx$

Put $\tan\sqrt{x} = t$

$\Rightarrow \frac{\sec^2\sqrt{x}}{2\sqrt{x}} dx = dt$

$\therefore\ I = \int \frac{\tan^4\sqrt{x} \cdot \sec^2\sqrt{x}}{\sqrt{x}} dx = \int 2t^4 dt$

$= \frac{2t^5}{5} + C = \frac{2(\tan\sqrt{x})^5}{5} + C$

Question 38

The slant height of a right circular cone is 3cm. The height of the cone for maximum volume is

Options:

A. 5cm

B. $\sqrt{3}$cm

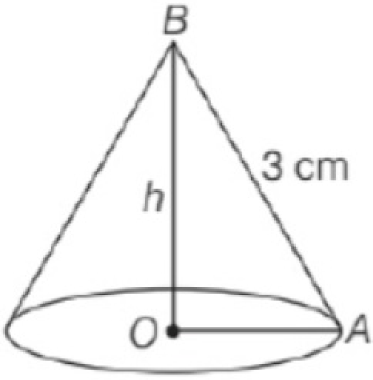
C. 3cm

D. $\sqrt{5}$cm

Answer: B

Solution:

Solution:
Let height of a right circular cone $= \text{hcm}$ and $\text{OA} = \text{rcm}$



Given, slant height of a right circular cone $= 3\text{cm}$ In $\triangle \text{OAB}$,
$\angle \text{BOA} = 90^\circ$
$(\text{OB})^2 + (\text{OA})^2 = (\text{AB})^2$
[apply pythogoras theorem]
$(\text{h})^2 + (\text{r})^2 = (3)^2$
$\text{r} = \sqrt{9 - \text{h}^2} \ldots$ (i)
We know that, Volume of cone
$= \frac{\pi}{3}\pi \text{r}^2 \text{h}$
$\text{V} = \frac{\pi}{3}(9 - \text{h}^2) \times \text{h}$
$\left[\text{ from Eq. (i), } \text{r} = \sqrt{9 - \text{h}^2} \right]$
$\text{V} = \frac{\pi}{3}(9\text{h} - \text{h}^3)$
$\frac{\text{dV}}{\text{dh}} = \frac{\pi}{3}(9 - 3\text{h}^2)$
$\therefore \frac{\text{dV}}{\text{dh}} = 0 \Rightarrow \frac{\pi}{3}(9 - 3\text{h}^2) = 0$
$\Rightarrow \frac{\text{h}^2}{} = \frac{9}{3} = 3 \Rightarrow \text{h} = \sqrt{3}$
$\frac{\text{d}^2\text{V}}{\text{dh}^2} = \frac{-6 \times \text{h}}{3} < 0$
So, $\text{h} = \sqrt{3}$ of the cone for maximum volume

Question 39

If $y = p[x(x - 2)]^2$ is an increasing function then the value of x is

Options:

A. 1, 2, 3

B. 0, 1, 2

C. 2,3

D. None of these

Answer: B

Solution:

Solution:

Given, function is $y = [x(x-2)]^2 = [x^2 - 2x]^2$
On differentiating both sides w.r.t. x, we get
$\frac{dy}{dx} = 2(x^2 - 2x)\frac{d}{dx}(x^2 - 2x)$
$= 2(x^2 - 2x)(2x - 2) = 4x(x-2)(x-1)$
On putting $\frac{dy}{dx} = 0$, we get
$4x(x-2)(x-1) = 0 \Rightarrow x = 0, 1$ and 2
Now, we find interval in which f (x) is strictly increasing or strictly decreasing.

Interval	$\frac{dy}{dx} = 4x(x-2)(x-1)$	Sign of $f'(x)$
$(-\infty, 0)$	$(-)(-)(-)$	$-ve$
$(0, 1)$	$(+)(-)(-)$	$+ve$
$(1, 2)$	$(+)(-)(+)$	$-ve$
$(2, \infty)$	$(+)(+)(+)$	$+ve$

Hence, y is strictly increasing in $(0, 1)$ and $(2, \infty)$. Also, y is a polynomial function, so it continuous at $X = 0, 1$ and 2 .
Hence, y is increasing in $[0, 1] \cup [2, \infty]$.

Question 40

If $y = \tan^{-1}\sqrt{\frac{1+\cos x}{1-\cos x}}$, then $\frac{dy}{dx}$ =

Options:

A. $\frac{3}{2}$

B. 0

C. 1

D. $-\frac{1}{2}$

Answer: D

Solution:

Solution:
$y = \tan^{-1}\sqrt{\frac{1+\cos x}{1-\cos x}}$
$y = \tan^{-1}\sqrt{\frac{2\cos^2\frac{x}{2}}{2\sin^2\frac{x}{2}}}$
$y = \tan^{-1}\left(\cot\frac{x}{2}\right)$
$y = \tan^{-1}\left[\tan\left(\frac{\pi}{2} - \frac{x}{2}\right)\right]$

$y = \frac{\pi}{2} - \frac{x}{2} \Rightarrow \frac{dy}{dx} = -\frac{1}{2}$

Question 41

If $\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ w.r.t. $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ is

Options:

A. $\frac{1}{2}$

B. $\frac{1}{4}$

C. $\frac{3}{2}$

D. $\frac{3}{4}$

Answer: B

Solution:

Solution:

Let $u = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$

Put $x = \tan\theta \Rightarrow \theta = \tan^{-1}x$, then

$u = \tan^{-1}\left[\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}\right] = \tan^{-1}\left[\frac{\sqrt{\sec^2\theta}-1}{\tan\theta}\right]$

$= \tan^{-1}\left[\frac{\sec\theta-1}{\tan\theta}\right] = \tan^{-1}\left[\frac{1-\cos\theta}{\sin\theta}\right]$

$= \tan^{-1}\left[\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right] = \tan^{-1}\left[\tan\frac{\theta}{2}\right]$

$\Rightarrow u = \frac{\theta}{2} = \frac{1}{2}\tan^{-1}x$

$r \cdot \tan^{-1}(\tan\theta) = \theta]$

On differentiating both sides w.r.t. x, we get

$\frac{du}{dx} = \frac{1}{2(1+x)^2}\left[\because \frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}\right] \ldots$ (i)

Also, let $v = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

Put $x = \tan\theta \Rightarrow \theta = \tan^{-1}x$, then we get

$v = \sin^{-1}\left[\frac{2\tan\theta}{1+\tan^2\theta}\right]$

$\Rightarrow v = \sin^{-1}[\sin 2\theta]$

$\Rightarrow v = 2\theta \Rightarrow v = 2\tan^{-1}x$

On differentiating both sides w.r.t. x, we get

$\frac{dv}{dx} = \frac{2}{1+x^2}$

Now, $\frac{du}{dv} = \frac{du}{dx} \times \frac{dx}{dv} = \frac{1}{2(1-x^2)} \times \frac{(1+x^2)}{2}$

[from Eqs. (i) and (ii)]

$\therefore \frac{du}{dv} = \frac{1}{4}$

Question 42

The region represented by the inequation system $x, y \geq 0, y \leq 6, x + y \leq 3$ is

Options:

A. unbounded in first quadrant

B. unbounded in first and second quadrants

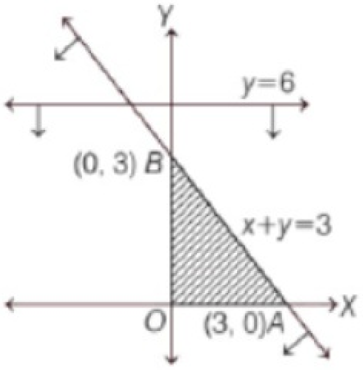
C. bounded in first quadrant

D. None of the above

Answer: C

Solution:

Solution:
The given region is bounded in first quadrant.



Question 43

The constraints $-x_1 + x_2 \leq 1, -x_1 + 3x_2 \leq 9$ and $x_1, x_2 \geq 0$ defines on

Options:

A. bounded feasible space

B. unbounded feasible space

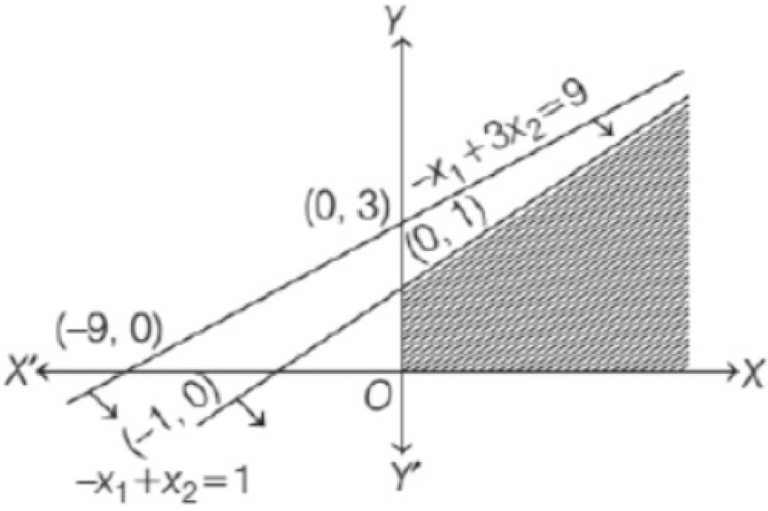
C. both bounded and unbounded feasible space

D. None of the above

Answer: B

Solution:

Solution:
It is clear from the figure that feasible space (shaded portion) is unbounded.



Question 44

If $z = \frac{(\sqrt{3}+i)^3(3i+4)^2}{(8+6i)^2}$, then |z| is equal to

Options:

A. 8

B. 2

C. 5

D. 4

Answer: B

Solution:

Solution:

Given, $z = \frac{(\sqrt{3}+i)^2(3i+4)^2}{(8+6i)^2}$

Now, $|z| = \left|\frac{(\sqrt{3}+i)^3(3i+4)^2}{(8+6i)^2}\right|$

$= \frac{|(\sqrt{3}+i)^3||(3i+4)^2|}{|(8+6i)^2|} \quad \left[\because \left|\frac{z_1}{z_2}\right| = \left|\frac{z_1}{z_2}\right|\right]$

$= \frac{|\sqrt{3}+i|^3 |3i+4|^2}{|8+6i|^2} \quad [\because |z^n| = |z|^n]$

$= \frac{(\sqrt{3+1})^3(\sqrt{9+16})^2}{(\sqrt{64+36})^2}$

$= \frac{(2)^3(5)^2}{(10)^2} = \frac{10^2 \cdot 2}{(10)^2} = 2$

Question 45

If $\frac{3}{2+\cos\theta+i\sin\theta}$ = a + ib, then $[(a-2)^2+b^2]$ is equal to

Options:

A. 0

B. 1

C. -1

D. 2

Answer: B

Solution:

Solution:

Given, $\frac{3}{2+\cos\theta+i\sin\theta} = a + ib$

$\Rightarrow \frac{3[(2+\cos\theta)-i\sin\theta]}{(2+\cos\theta)^2+\sin^2\theta} = a + ib$

$\Rightarrow \frac{3[2+\cos\theta-i\sin\theta]}{5+4\cos\theta} = a + ib$

$\Rightarrow a = \frac{3(2+\cos\theta)}{5+4\cos\theta}$ and $b = -\frac{3\sin\theta}{5+4\cos\theta}$

$\therefore (a-2)^2+b^2 = \left(\frac{6+3\cos\theta}{5+4\cos\theta}-2\right)^2 + \frac{9\sin^2\theta}{(5+4\cos\theta)^2}$

$= \frac{(-4-5\cos\theta)^2+9\sin^2\theta}{(5+4\cos\theta)^2}$

$= \frac{16+25\cos^2\theta+40\cos\theta+9\sin^2\theta}{(5+4\cos\theta)^2}$

$= \frac{16+16\cos^2\theta+40\cos\theta+9}{(5+4\cos\theta)^2}$

$= \frac{(5+4\cos\theta)^2}{(5+4\cos\theta)^2} = 1$

Question 46

A box contains 100 bulbs, out of which 10 are defective. A sample of 5 bulbs is drawn. The probability that none is defective, is

Options:

A. $\frac{9}{10}$

B. $\left(\frac{1}{10}\right)^5$

C. $\left(\frac{9}{10}\right)^5$

D. $\left(\frac{1}{2}\right)^5$

Answer: C

Solution:

Solution:

Let probability of defective bulb,

$p = \frac{10}{100} = \frac{1}{10} = 0.1$

and probability of non-defective bulb,

$q = 1 - 0.1 = 0.9$

Here, $n = 5$

$\therefore P(\text{ none is defective }) = P(X = 0)$

$= {}^5C_0(0.1)^0(0.9)^5$

$= 1 \times (0.9)^5 = \left(\frac{9}{10}\right)^5$

Question 47

A random variable X has the probability distribution given below.

X	1	2	3	4	5
P(x=X)	K	2K	3K	2K	K

Its variance is

Options:

A. $\frac{16}{3}$

B. $\frac{4}{3}$

(C) $\frac{5}{3}$

C. $\frac{10}{3}$

Answer: B

Solution:

Solution:

X	1	2	3	4	5
P(X=x)	K	2K	3K	2K	K

$\therefore$ Variance $= \Sigma x_i^2 p - (\Sigma x_i p)^2$

$= (1k + 8k + 27k + 32k + 25k)$

$- (k + 4k + 9k + 8k + 5k)^2$

$= (93k) - (27k)^2 = \left(93 \times \frac{1}{9}\right) - \left(27 \times \frac{1}{9}\right)^2$

$= \frac{93}{9} - 9 = \frac{93 - 81}{9} = \frac{12}{9} = \frac{4}{3}$

Question 48

The area (in sq. units) of the region bounded by the curves $y^2 = 4ax$ and $x^2 = 4ay$, $a > 0$ is

Options:

A. $\frac{16a^2}{3}$

B. $\frac{14a^2}{3}$

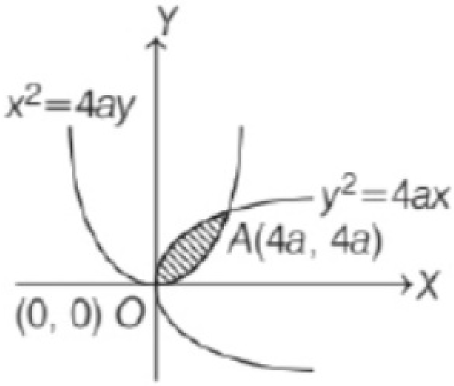
C. $\frac{13a^2}{3}$

D. $16a^2$

Answer: A

Solution:

Solution:
The equations of given curves are
$y^2 = 4ax$ and $x^2 = 4ay$



On solving these equations, we get the intersection points, i.e. (0, 0) and (4a, 4a).

$\therefore$ Required area $= \int_0^{4a} \left(2\sqrt{a}\sqrt{x} - \frac{x^2}{4a}\right) dx$

$= 2\sqrt{a}\left[\frac{x^{3/2}}{3/2}\right]_0^{4a} - \left[\frac{x^3}{12a}\right]_0^{4a}$

$= \frac{32a^2}{3} - \frac{16a^2}{3}$

$= \frac{16a^2}{3}$

Question 49

The area in the positive quadrant enclosed by the circles $x^2 + y^2 = 4$, the line $x = y\sqrt{3}$ and X -axis is

Options:

A. $\frac{\pi}{2}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

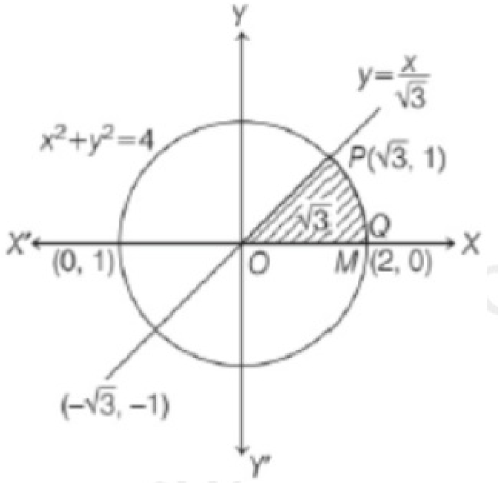
D. π

Answer: C

Solution:

Solution:

The intersection points of curves $x^2 + y^2 = 4$ and $y = \frac{x}{\sqrt{3}}$ are $(0, 0)$ and $P(\sqrt{3}, 1)$.



$\therefore$ Area of $\triangle OPM = \frac{1}{2} \times \sqrt{3} \times 1 = \frac{\sqrt{3}}{2}$

and area of curve $MPQ = \int_{\sqrt{3}}^{2} \sqrt{4 - x^2} dx$

$= \left[\frac{x}{2}\sqrt{4 - x^2} + \frac{4}{2}\sin^{-1}\left(\frac{x}{2} \right) \right]_{\sqrt{3}}^{2}$

$= \left[0 + 2\left(\frac{\pi}{2} \right) - \left(\frac{\sqrt{3}}{2} + 2 \times \frac{\pi}{3} \right) \right]$

$= \left(\frac{\pi}{3} - \frac{\sqrt{3}}{2} \right)]$

Question 50

$\int_{\pi/6}^{\pi/2} \frac{\text{cosec}x \cdot \cot x}{1 + \text{cosec}^2 x} \mathbf{d\,x} =$

Options:

A. $\tan^{-1}2$

B. $\tan^{-1}\left(\frac{1}{3} \right)$

C. $\tan^{-1}1$

D. $\frac{\pi}{4} - \tan^{-1}2$

Answer: B

Solution:

Solution:
Let $I = \int_{\pi/6}^{\pi/2} \frac{\text{cosec}x \cdot \cot x}{1 + \text{cosec}^2 x} dx$
Let cosecx = t
$\Rightarrow -\text{cosec}x \cot x \, dx = dt$
When $x = \frac{\pi}{6}$, then $t = \text{cosec} \frac{\pi}{6} = 2$

and when $x = \frac{\pi}{2}$, then $t = \text{cosec} \frac{\pi}{2} = 1$

$\therefore I = \int_2^1 -\frac{dt}{1 + t^2} = -\int_2^1 \frac{dt}{1 + t^2}$

$= \int_1^2 \frac{dt}{1 + t^2} = [\tan^{-1}(t)]_1^2$

$= \tan^{-1}(2) - \tan^{-1}(1)$

$= \tan^{-1}\left(\frac{2 - 1}{1 + 2 \times 1}\right) = \tan^{-1}\left(\frac{1}{1 + 2}\right)$

$= \tan^{-1}\left(\frac{1}{3}\right)$

Question 51

If $\sin A + \cos A = \sqrt{2}$, then the value of $\cos^2 A$ is

Options:

A. $\sqrt{2}$

B. $\frac{1}{2}$

C. 4

D. -4

Answer: B

Solution:

Solution:
Given, $\sin A + \cos A = \sqrt{2}$

$\therefore \frac{1}{\sqrt{2}} \sin A + \frac{1}{\sqrt{2}} \cos A = 1$

$\Rightarrow \sin \frac{\pi}{4} \sin A + \cos A \cos \frac{\pi}{4} = 0$

$\Rightarrow \cos\left(A - \frac{\pi}{4}\right) = 1$

$\Rightarrow A - \frac{\pi}{4} = 0$

$\Rightarrow A = \frac{\pi}{4}$

Now, $\cos^2 A = \cos^2 \frac{\pi}{4} = \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2}$

Question 52

The number of solutions of equation $\tan x + \sec x = 2\cos x$ lying in the interval $[0, 2\pi]$ is

Options:

A. 0

B. 1

C. 2

D. 3

Answer: C

Solution:

Solution:
$\tan x + \sec x = 2\cos x$
$\frac{\sin x}{\cos x} + \frac{1}{\cos x} = 2\cos x$
$\Rightarrow 1 + \sin x = 2\cos^2 x$
$\Rightarrow 1 + \sin x = 2(1 - \sin^2 x)$
$\Rightarrow 1 + \sin x = 2 - 2\sin^2 x$
$\Rightarrow 2\sin^2 x + \sin x - 1 = 0$
$\Rightarrow 2\sin^2 x + 2\sin x - \sin x - 1 = 0$
$\Rightarrow 2\sin x(\sin x + 1) - 1(\sin x + 1) = 0$
$\Rightarrow (\sin x + 1)(2\sin x - 1) = 0$
$\Rightarrow \sin x + 1 = 0 \Rightarrow 2\sin x - 1 = 0$
$\Rightarrow \sin x = -1, \sin x = \frac{1}{2}$
$\Rightarrow x = \frac{3\pi}{2}, x = \frac{\pi}{6}$

Question 53

The value of $\tan 75^\circ - \cot 75^\circ$ is

Options:

A. $2\sqrt{3}$

B. $2 - \sqrt{3}$

C. $2 + \sqrt{3}$

D. 0

Answer: A

Solution:

Solution:
$\tan 75^\circ - \cot 75^\circ$

$$= \frac{\sin 75^\circ}{\cos 75^\circ} - \frac{\cos 75^\circ}{\sin 75^\circ}$$
$$= \frac{\sin^2 75^\circ - \cos^2 75^\circ}{\sin 75^\circ \cdot \cos 75^\circ} = \frac{-2\cos 150^\circ}{\sin 150^\circ}$$
$$= \frac{-2\cos(90^\circ + 60^\circ)}{\sin(90 + 60^\circ)}$$
$$= \frac{2 \cdot \frac{\sqrt{3}}{2}}{\frac{1}{2}} = 2\sqrt{3}$$

Question 54

$\cos^{-1}\alpha + \cos^{-1}\beta + \cos^{-1}\gamma = 3\pi$, then $\alpha(\beta + \gamma) + \beta(\gamma + \alpha) + \gamma(\alpha + \beta)$ equals

Options:

A. 8

B. 1

C. 6

D. 12

Answer: C

Solution:

Solution:
$\cos^{-1}\alpha + \cos^{-1}\beta + \cos^{-1}\gamma = 3\pi$
$0 \le \cos^{-1}x \le \pi$
$\cos^{-1}\alpha + \cos^{-1}\beta + \cos^{-1}\gamma = 3\pi$
$\cos^{-1}\alpha = \cos^{-1}\beta = \cos^{-1}\gamma = \pi$
$\cos\pi = \alpha = \beta = \gamma$
$-1 = \alpha = \beta = \gamma$
$\alpha = \beta = \gamma = -1$
$\alpha(\beta + \gamma + \beta(\gamma + \alpha) + \gamma(\alpha + \beta)$
$-1(-1-1) - 1(-1-1) - 1(-1-1)$
$= 2 + 2 + 2 = 6$

Question 55

A line has slope m and y-intercept 4. The distance between the origin and the line is equal to

Options:

A. $\frac{4}{\sqrt{1-m^2}}$

B. $\frac{4}{\sqrt{m^2 - 1}}$

C. $\frac{4}{\sqrt{m^2 + 1}}$

D. $\frac{4m}{\sqrt{1 + m^2}}$

Answer: C

Solution:

Solution:
Equation of line is $y = mx + 4$
$\therefore$ Required distance $= \frac{4}{\sqrt{1 + m^2}}$

Question 56

If a line with y-intercept 2 , is perpendicular to the line 3x − 2y = 6, then its x-intercept is

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Options:

A. 1

B. 2

C. −4

D. 3

Answer: D

Solution:

Solution:
Let the equation of perpendicular line to the line
$3x - 2y = 6$ be $3y + 2x = c$
Since, it passes through (0, 2).
$\therefore \; c = 6$
On putting the value of c in Eq. (i), we get
$\Rightarrow 3y + 2x = 6$
$\frac{x}{3} + \frac{y}{2} = 1$
Hence, x-intercept is 3 .

Question 57

A line passes through the point of intersection of the lines 3x + y + 1 = 0 and 2x − y + 3 = 0 and makes equal intercepts with axes. Then, equation

of the line is

Options:

A. $5x + 5y - 3 = 0$

B. $x + 5y - 3 = 0$

C. $5x - y - 3 = 0$

D. $5x + 5y + 3 = 0$

Answer: A

Solution:

Solution:

The point of intersection of the lines $3x + y + 1 = 0$ and $2x - y + 3 = 0$ is $\left(-\frac{4}{5}, \frac{7}{5}\right)$

The equation of line, which makes equal intercepts with axes, is $x + y = a$.

$\therefore \; -\frac{4}{5} + \frac{7}{5} = a \Rightarrow a = \frac{3}{5}$

Now, equation of line is $x + y - \frac{3}{5} = 0$

$\Rightarrow \; 5x + 5y - 3 = 0$

Question 58

If the circles $x^2 + y^2 = 9$ and $x^2 + y^2 + 2\alpha x + 2y + 1 = 0$ touch each other internally, then α is equal to

Options:

A. $\pm \frac{4}{3}$

B. 1

C. $\frac{4}{3}$

D. $-\frac{4}{3}$

Answer: A

Solution:

Solution:

Centres and radii of the given circles are $C_1(0, 0)$, $r_1 = 3$ and $C_2(-\alpha, -1)$

$r_2 = \sqrt{\alpha^2 + 1 - 1} = |\alpha|$

Since, two circles touch internally.

$\therefore C_1C_2 = r_1 - r_2$

$\Rightarrow \sqrt{\alpha^2 + 1^2} = 3 - |\alpha|$

$\Rightarrow \alpha^2 + 1 = 9 + \alpha^2 - 6|\alpha|$

$\Rightarrow 6|\alpha| = 8 \Rightarrow |\alpha| = \frac{4}{3}$

$\therefore \alpha = \pm\frac{4}{3}$

Question 59

The length of the common chord of the two circles $x^2 + y^2 - 4y = 0$ and $x^2 + y^2 - 8x - 4y + 11 = 0$ is

Options:

A. $\frac{\sqrt{145}}{4}$cm

B. $\frac{\sqrt{11}}{2}$cm

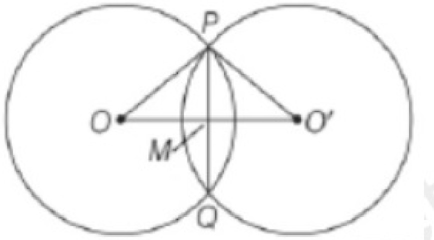
C. $\sqrt{135}$cm

D. $\frac{\sqrt{135}}{4}$cm

Answer: D

Solution:

Solution:
Given, equation of circles are $x^2 + y^2 - 4y = 0$ and $x^2 + y^2 - 8x - 4y + 11 = 0$
∴ Equation of chord is
$x^2 + y^2 - 4y - (x^2 + y^2 - 8x - 4y + 11) = 0$
$\Rightarrow 8x - 11 = 0$



So, centre and radius of first circle are O(0, 2) and OP = r = 2.
Now, perpendicular distance from O(0, 2) to the line 8x − 11 is

$d = OM = \frac{|8 \times 0 - 11|}{\sqrt{8^2}} = \frac{11}{8}$

In ΔOMP, $PM = \sqrt{OP^2 - OM^2}$

$= \sqrt{2^2 - \left(\frac{11}{8}\right)^2} = \sqrt{4 - \frac{121}{64}}$

$= \sqrt{\frac{256 - 121}{64}} = \frac{\sqrt{135}}{8}$

∴ Length of chord $PQ = 2PM$

$= 2 \times \frac{\sqrt{135}}{8} = \frac{\sqrt{135}}{4}$cm

Question 60

If $x_1, x_2, \ldots, x_{18}$ are observations such that $\sum_{j=1}^{18}(x_j - 8) = 9$ and $\sum_{j=1}^{18}(x_j - 8)^2 = 45$, then standard deviation of these observations is

Options:

A. $\sqrt{\frac{81}{34}}$

B. 5

C. $\sqrt{5}$

D. $\frac{3}{2}$

Answer: D

Solution:

Solution:
Standard deviation

$$\sqrt{\frac{\sum_{j=1}^{18}(x_j - 8)^2}{n} - \left(\frac{\sum_{j=1}^{18}(x_j - 8)}{n}\right)^2}$$

$$= \sqrt{\frac{45}{18} - \left(\frac{9}{18}\right)^2} = \sqrt{\frac{45}{18} - \frac{1}{4}}$$

$$= \sqrt{\frac{81}{36}} = \frac{9}{6} = \frac{3}{2}$$

Question 61

The sum of the deviations of the variates from the arithmetic mean is always

Options:

A. +1

B. 0

C. −1

D. real number

Answer: B

Solution:

Solution:
Let $x_1, x_2, \ldots, x_n$ be n variates. Then, their arithmetic mean will be

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} \quad \ldots \text{(i)}$$

Now, the sum of deviation of the variates from the AM, i.e. SD $(\bar{x})$

$$= \sum_{i=1}^{n} (x_i - \bar{x}) = \sum_{i=1}^{n} x_i - \sum_{i=1}^{n} \bar{x}$$

$$= \sum_{i=1}^{n} x_i - n\bar{x} = \sum_{i=1}^{n} x_i - \sum_{i=1}^{n} x_i \quad \text{[from Eq. (i)]}$$

$$= 0$$

Question 62

A student answers a multiple choice question with 5 alternatives, of which exactly one is correct. The probability that he knows the correct answer is p, 0 < p < 1. If he does not know the correct answer, he randomly ticks one answer. Given that he has answered the question correctly, the probability that he did not tick the answer randomly, is

Options:

A. $\frac{3p}{4p+3}$

B. $\frac{5p}{3p+2}$

C. $\frac{5p}{4p+1}$

D. $\frac{4p}{3p+1}$

Answer: C

Solution:

Solution:

Let E_1 = Student does not know the answer E_2 = Student knows the answer

and E = student answer correctly

$\therefore P(E_1) = 1 - p \Rightarrow P(E_2) = p$

$\Rightarrow P\left(\frac{E}{E_2}\right) = 1$ and $P\left(\frac{E}{E_1}\right) = \frac{1}{5}$

Note that, the probability that student did not know the answer randomly = The probability that student know the answer.

$$\therefore P\left(\frac{E_2}{E}\right) = \frac{P(E_2)P\left(\frac{E}{E_2}\right)}{P(E_1)P\left(\frac{E}{E_1}\right) + P(E_2)P\left(\frac{E}{E_2}\right)}$$

$$= \frac{p(1)}{(1-p)\frac{1}{5} + p(1)} = \frac{p}{\frac{1-p+5p}{5}}$$

$$= \frac{5p}{1+4p}$$

Question 63

Box A contains 2 black and 3 red balls, while box B contains 3 black and 4 red balls. Out of these two boxes one is selected at random and the probability of choosing box A is double that of box B.
If a red ball is drawn from the selected box, then the probability that it has come from box B, is

Options:

A. $\frac{21}{41}$

B. $\frac{10}{31}$

C. $\frac{12}{31}$

D. $\frac{13}{41}$

Answer: B

Solution:

Solution:
Let probability of choosing box B, P(B) = p
According to the given condition,
$P(A) = 2P(B) = 2p$

$$\text{Now, } P\left(\frac{R}{A}\right) = \frac{{}^3C_1}{{}^5C_1} = \frac{3}{5}$$

$$\text{and } P\left(\frac{R}{B}\right) = \frac{{}^4C_1}{{}^7C_1} = \frac{4}{7}$$

$$\therefore P\left(\frac{B}{R}\right) = \frac{P(B)\cdot P\left(\frac{R}{B}\right)}{P(A)\cdot P\left(\frac{R}{A}\right) + P(B)\cdot P\left(\frac{R}{B}\right)}$$

$$= \frac{p\cdot\frac{4}{7}}{2p\cdot\frac{3}{5} + p\cdot\frac{4}{7}} = \frac{10}{31}$$

Question 64

The contrapositive of the statement. 'If $2^2 = 5$, then I get first class' is

Options:

A. If I do not get a first class, then $2^2 = 5$

B. If I do not get a first class, then $2^2 \neq 5$

C. If I get a first class, then $2^2 = 5$

D. None of the above

Answer: B

Solution:

Solution:
Let p and q be two propositions given by $p : 2^2 = 5$, q: I get first class.
Then, given statement is $p \rightarrow q$.
The contrapositive of this statement is
$\sim q \rightarrow \sim p$, i.e. if I do not get first class, then $2^2 \neq 5$.

Question 65

The truth value of the statement 'Patna is in Bihar or $5 + 6 = 111'$ is

Options:

A. true

B. false

C. Cannot say anything

D. None of these

Answer: A

Solution:

Solution:
Let p : Patna is in Bihar and $q : 5 + 6 = 111$
Then, the given statement is disjunction $p \vee q$.
Since, p is true and q is false.
∴ The disjunction $p \vee q$ is true.
Hence, truth value of given statement is true.

Question 66

If $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$, then A^3 is

Options:

A. 3A

B. 2A

C. 4A

D. A

Answer: C

Solution:

Solution:

We have, $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

Now, $A^2 = A \cdot A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1+1 & -1-1 \\ -1-1 & 1+1 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$

and $A^3 = A^2 \cdot A = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 2+2 & -2-2 \\ -2-2 & 2+2 \end{bmatrix} \begin{bmatrix} 4 & -4 \\ -4 & 4 \end{bmatrix}$

$\therefore\ A^3 = 4A$

Question 67

If $A = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}$ and A adj A = AA^T, then 5a + b is equal to

Options:

A. 5

B. 4

C. 13

D. −1

Answer: A

Solution:

Solution:

$A = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}$ and $A \text{ adj } A = AA^T$

$\therefore\ \text{adj } A = \begin{bmatrix} 2 & b \\ -3 & 5a \end{bmatrix}$

Now, $AA^T = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 5a & 3 \\ -b & 2 \end{bmatrix}$

$= \begin{bmatrix} 25a^2 + b^2 & 15a - 2b \\ 15a - 2b & 13 \end{bmatrix}$

and $A \cdot \text{ad jA} = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 2 & b \\ -3 & 5a \end{bmatrix}$

$$= \begin{bmatrix} 10a + 3b & 0 \\ 0 & 10a + 3b \end{bmatrix}$$

$\because A \cdot (\text{ad jA}) = AA^T$ is given, so equating the two expression, we get

$$\begin{bmatrix} 25a^2 + b^2 & 15a - 2b \\ 15a - 2b & 13 \end{bmatrix} = \begin{bmatrix} 10a + 3b & 0 \\ 0 & 10a + 3b \end{bmatrix}$$

We have, $10a + 3b = 13$ and $15a - 2b = 0$
On solving, we get
$a = \frac{2}{5}$ and $b = 3$
$\Rightarrow 5a + b = 5 \times \frac{2}{3} + 3$
$\Rightarrow 5a + b = 2 + 3$
$\Rightarrow 5a + b = 5$

Question 68

The value of f at x = 0, so that function $f(x) = \frac{2^x - 2^{-x}}{x}$, $x \neq 0$ is continuous at x = 0, is

Options:

A. 0

B. $\log 4$

C. 4

D. e^4

Answer: B

Solution:

Solution:

$\lim_{x \to 0} \frac{2^x - 2^{-x}}{x} = \lim_{x \to 0} 2^x \log 2 + 2^{-x} \log 2$
[using L'Hospital's rule]
$= \log 2 + \log 2 = \log 4$
Since, function is continuous at $x = 0$.
$\therefore\ f(0) = \lim_{x \to 0} \frac{2^x - 2^{-x}}{x} = \log 4$

Question 69

If $f(x) = \begin{cases} ax + 3 & x \le 2 \\ a^2x - 1 & x > 2. \end{cases}$, then the values of a for which f is continuous for all x are

Options:

A. 1 and -2

B. 1 and 2

C. -1 and 2

D. -1 and -2

Answer: C

Solution:

Solution:

Given, f (x) = begin cases a x + 3, x ≤ 2\a^{2}x − 1, x > 2 end cases

Continuity at $x = 2$,

$$\text{LHL} = \lim_{x \to 2^-} f(x) = \lim_{x \to 2} (ax + 3) = 2a + 3$$

$$\text{RHL} = \lim_{x \to 2^+} f(x) = \lim_{x \to 2} (a^2x - 1) = 2a^2 - 1$$

Since, f (x) is continuous for all values of x.

$\therefore\ \text{LHL} = \text{RHL}$

$\Rightarrow 2a + 3 = 2a^2 - 1$

$\Rightarrow 2a^2 - 2a - 4 = 0$

$\Rightarrow a^2 - a - 2 = 0$

$\Rightarrow a^2 - 2a + a - 2 = 0$

$\Rightarrow a(a - 2) + 1(a - 2) = 0$

$\Rightarrow (a + 1)(a - 2) = 0$

$\therefore\ a = -1, 2$

Question 70

$\lim\limits_{x \to 0} \dfrac{1 - \cos^3 x}{x \sin x \cos x}$ **is equal to**

Options:

A. $\frac{2}{5}$

B. $\frac{3}{5}$

C. $\frac{3}{2}$

D. $\frac{3}{4}$

Answer: C

Solution:

Solution:

$$\lim_{x \to 0} \frac{1 - \cos^3 x}{x \sin x \cos x}$$

$$= \lim_{x \to 0} \frac{(1 - \cos x)(1 + \cos^2 x + \cos x)}{x^2 \cos x \cdot \frac{\sin x}{x}}$$

$$= 3\lim_{x \to 0} \frac{1 - \cos x}{x^2} = 3 \times \frac{1}{2} = \frac{3}{2}$$

Question 71

The value of $\lim_{x \to 0} \left(\frac{a^x + b^x + c^x}{3} \right)^{2/x}$, (a, b, c > 0) is

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Options:

A. $(abc)^3$

B. abc

C. $(abc)^{1/3}$

D. None of these

Answer: D

Solution:

Solution:

Let $y = \lim_{y \to 0} \left(\frac{a^x + b^x + c^x}{3} \right)^{2/x}$

$\Rightarrow \log y = \lim_{x \to 0} \frac{2}{x} \log \left(\frac{a^x + b^x + c^x}{3} \right)$

$\Rightarrow \log y = \lim_{x \to 0} \frac{2 \log(a^x + b^x + c^x) - \log 3}{x}$

$\Rightarrow \log y = \log(abc)^{2/3}$

$\therefore$ [using L'Hospital's rule]

$\therefore y = (abc)^{2/3}$

Question 72

If the vectors $2\hat{i} - \hat{j} - \hat{k}$, $\hat{i} + 2\hat{j} - 3\hat{k}$ and $3\hat{i} + \lambda\hat{j} + 5\hat{k}$ are coplanar, then the value of λ is

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Options:

A. −8

B. −4

C. −2

D. −1

Answer: B

Solution:

Solution:

Let $a = 2\hat{i} - \hat{j} + \hat{k}$, $b = \hat{i} + 2\hat{j} - 3\hat{k}$,

$c = 3\hat{i} + \lambda\hat{j} + 5\hat{k}$

Now given vectors a, b, c will be coplanar if

$a \cdot (b \times c) = 0$, i.e. $[ac\ b\ \ c| = 0$

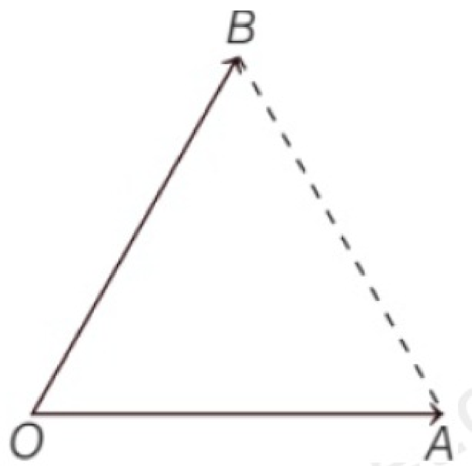
$$\begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & -3 \\ 3 & \lambda & 5 \end{vmatrix} = 0$$

$\Rightarrow 2(10 + 3\lambda) + 1(5 + 9) + 1(\lambda - 6) = 0$

$\Rightarrow\ 7\lambda = -28 = 0 \Rightarrow \lambda = -4$

Question 73

A $\triangle$OAB is determined by the vector a and b as show in the figure what will be area of triangle?



Options:

A. $\frac{1}{2}\sqrt{|a|^2 + |\,b\,|^2 - (a \cdot b)}$

B. $\frac{1}{2}\sqrt{|a|\,|\,b\,| - (a \cdot b)^2}$

C. $\frac{1}{4}\sqrt{|a|^2 + |\,b\,|^2 - (a \cdot b)}$

D. $\frac{1}{4}\sqrt{|a|\,|\,b\,| - (a \cdot b)^2}$

Answer: B

Solution:

Solution:

We know that area of triangle $= \frac{1}{2}\left|OA \times OB\right|$

$\Delta^2 = \frac{1}{4}\left|a \times b\right| \ldots$ (i)

Now $|a \times b|^2 + (a \cdot b)^2$

$= |\,a|^2\,|\,b\,|^2 \sin^2\theta + |\,a|^2\,|\,b\,|^2 \cos^2\theta$

$= |a|^2 |b|^2 (\sin^2\theta + \cos^2\theta)$
$= |a|^2 |b|^2 \times 1 = |a|^2 |b|^2$
$\Rightarrow |a \times b|^2 = |a|^2 |b|^2 - (a \cdot b)^2$
From Eq. (i), we get
$\Delta^2 = \frac{1}{4}[|a|^2 |b|^2 - (a \cdot b)^2]$
$\Delta = \frac{1}{2}\sqrt{|a|^2 + |b|^2 - (a \cdot b)^2}$

Question 74

The line $\frac{x-2}{3} - \frac{y-1}{-5} = \frac{z+2}{2}$ lies in the plane $x + 3y - \alpha z + \beta = 0$, then value of $\alpha\beta$ is

Options:

A. -42

B. 1

C. -2

D. 42

Answer: A

Solution:

Solution:
Given equation of line
$\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{2} \ldots$ (i)
The direction ratios of the normal are $(1, 3, -\alpha)$.
The direction ratios of the line are $(3, -5, 2)$ and equation of given plane
$x + 3y - az + \beta = 0 \ldots$ (ii)
Four lines are perpendicular
$\Rightarrow a_1a_2 + b_1b_2 + c_1c_2 = 0$
$\Rightarrow 3 - 15 + 2\alpha = 0$
$\Rightarrow 2\alpha = -12 \Rightarrow \alpha = -6$
$(2, 1, -2)$ lies on the plane, so
$2 + 3 + 6(-2) + \beta = 0 \Rightarrow \beta = 7$
$\alpha \cdot \beta = -6 \times 7 = -42$

Question 75

Find the angle between the lines whose direction cosines are given by the equations $3l + m + 5n = 0$, $6mn - 2nl + 5lm = 0$

Options:

A. $\cos^{-1}\frac{1}{6}$

B. $\cos^{-1}\frac{2}{6}$

C. $\sin^{-1}\frac{1}{6}$

D. $\sin^{-1}\frac{2}{6}$

Answer: A

Solution:

Solution:
The given equations are $3l + m + 5n = 0$ . . .(i)
and $6mn - 2nl + 5/m = -0$. . . (ii)
Now, from Eq. (i), we get
$m = -3l - 5n$
On substituting $m = -3/-5n$ in Eq. (ii), we get
$6(-3l - 5n)n - 2nl + 5l(-3l - 5n) = 0$
$\Rightarrow 30n^2 + 45ln + 15I^2 = 0$
$\Rightarrow 2n^2 + 3ln + I^2 = 0$
$\Rightarrow 2n^2 + 2nl + nl + I^2 = 0$
$\Rightarrow 2n(n + l) + l(n + l) = 0$
$\Rightarrow (n + l)(2n + l) = 0$
$\Rightarrow$ Either $I = -n$ or $I = -2n$
If $I = -n$, then $m = -2n$ and if $I = -2n$, then $m = n$
Thus, the direction ratios of two lines are proportional $(-n, -2n, n)$ and $(-2n, n, n)$ i.e. $(-1, -2, 1)$ and $(-2, 1, 1)$, respectively.
Now, let θ be the acute angle between the lines,

$$\text{Then, } \cos\theta = \frac{|a_1a_2 + b_1b_2 + c_1c_2|}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$$= \frac{|2 - 2 + 1|}{\sqrt{1 + 4 + 1}\sqrt{4 + 1 + 1}} = \frac{1}{6}$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{1}{6}\right)$$

Question 76

Find the shortest distance between the lines $r = \hat{i} + \hat{j} + \lambda\left(2\hat{i} - \hat{j} + \hat{k}\right)$ and $r = 2\hat{i} + \hat{j} - \hat{k} + \mu\left(3\hat{i} - 5\hat{j} + 2\hat{k}\right)$.

Options:

A. $\frac{10}{\sqrt{59}}$

B. $\frac{8}{\sqrt{57}}$

C. $\frac{9}{\sqrt{89}}$

D. $\frac{10}{\sqrt{39}}$

Answer: A

Solution:

Solution:

$r = (\hat{i} + \hat{j}) + \lambda(2\hat{i} - \hat{j} + \hat{k})$ . . . (i)

$r = (2\hat{i} + \hat{j} - \hat{k}) + \mu(3\hat{i} - 5\hat{j} + 2\hat{k})$ . . . (ii)

Compare with vector equation $r = a + \lambda b$

$a_1 = \hat{i} + \hat{j} = 2\hat{i} - \hat{j} + \hat{k}$

$a_2 = 2\hat{i} + \hat{j} - \hat{k}, b_2 = 3\hat{i} - 5\hat{j} + 2\hat{k}$

$d = \left| \frac{(b_1 \times b_2) \cdot (a_2 - a_1)}{|b_1 \times b_2|} \right|$ . . . (iii)

$$b_1 \times b_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 3 & -5 & 2 \end{vmatrix}$$

$b_1 \times b_2 = \hat{i}(-2 + 5) - \hat{j}(4 - 3) + \hat{k}(-10 + 3)$

$= 3\hat{i} - \hat{j} - 7\hat{k}$

$|b_1 \times b_2| = \sqrt{(3)^2 + (-1)^2 + (-7)^2}$

Also $a_2 - a_1 = (2\hat{i} + \hat{j} - \hat{k}) + (\hat{i} + \hat{j}) = \hat{i} - \hat{k}$... (iv)

$d = \left| \frac{(3\hat{i} - \hat{j} - 7\hat{k})(\hat{i} - \hat{k})}{\sqrt{59}} \right| = \left| \frac{3 - 0 + 7}{\sqrt{59}} \right| = \frac{10}{\sqrt{59}}$

Question 77

$\int \frac{dx}{\sin^2 x \cos^2 x}$ **is equal to**

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Options:

A. $\tan x + \cot x + C$

B. $\tan x - \cot x + C$

C. $\tan x - \cot^2 x + C$

D. None of these

Answer: B

Solution:

Solution:

$I = \int \frac{dx}{\sin^2 x \cos^2 x}$

$= \int \frac{(\sin^2 x + \cos^2 x)}{\sin^2 x \cdot \cos^2 x}$ $[\because \sin^2\theta + \cos^2\theta = 1]$

$= \int (\sec^2 x + \operatorname{cosec}^2 x) dx$

$= \int (\sec^2 x dx + \int \operatorname{cosec}^2 x dx$

$= \tan x - \cot x + C$

Question 78

$\int \frac{e^{\tan^{-1}x}}{1+x^2} dx$ **is equal to**

Options:

A. $\frac{\tan^{-1}x}{x} + C$

B. $\frac{\tan^{-1}x}{x} + C$

C. $e^{\tan^{-1}x} + C$

D. $\frac{e^{\tan^{-1}x}}{x^2} + C$

Answer: C

Solution:

Solution:

Let $I = \int \frac{e^{\tan^{-1}x}}{1+x^2} dx$

Put $\tan^{-1}x = t$

$\Rightarrow \frac{1}{1+x^2} dx = dt$

$\therefore I = \int \frac{e^{\tan^{-1}x}}{1+x^2} dx = \int e^t dt$

$= e^t + C \ [\because \int e^x dx = e^x]$

$= e^{\tan^{-1}x} + C \quad [\text{put } t = \tan^{-1}x]$

Question 79

A spherical raindrop evaporates at a rate proportional to its surface originally is 3mm and 1h later has been reduced to 2mm, then radius r of the raindrop at any time t is (where $0 \le t < 3$)

Options:

A. $r = t + 3$

B. $r = t + 5$

C. $r = t - 5$

D. $r = 3 - t$

Answer: D

Solution:

Solution:

Let r be the radius, V be the volume and S be the surface area of the spherical raindrop at time t.

Then, $V = \frac{4}{3}\pi r^3$ and $S = 4\pi r^2$

The rate at which the raindrop evaporates is $\frac{dV}{dt}$

which is proportional to the surface area.

$\therefore \frac{dV}{dt} \propto S \Rightarrow \frac{dV}{dt} = -kS, \quad \text{where } k > 0 \ldots$ (i)

Now, $V = \frac{4}{3}\pi r^3$ and $S = 4\pi r^2$

$\therefore \frac{dV}{dt} = \frac{4\pi}{3} \times 3r^2 \frac{dr}{dt} = 4\pi r^2 \frac{dr}{dt}$

$4\pi r^2 \frac{dr}{dt} = -k(4\pi r^2)$ [from Eq. (i)]

$\frac{dr}{dt} = -k \Rightarrow dr = -k\,dt$

On integrating, we get

$\int dr = -k \int dt + C$

$\therefore r = -kt + C$

Initially, i.e. when $t = 0, r = 3$

$\therefore 3 = -k \times 0 + C$

$\therefore C = 3$

$\therefore r = -kt + 3$

When $t = 1, r = 2$

$\therefore 2 = -k \times 1 + 3$

$\therefore k = 1$

$\therefore r = -t + 3$

$\therefore r = 3 - t$, where $0 \le t \le 3$

This is the required expression for the radius of the raindrop at any time t.

Question 80

If $y = \sin^{-1}\left(6x\sqrt{1-9x^2}\right), -\frac{1}{3\sqrt{2}} < x < \frac{1}{3\sqrt{2}}$ then $\frac{dy}{dx}$ is

Options:

A. $\frac{6}{\sqrt{1-9x^2}}$

B. $\frac{5}{\sqrt{1-3x^2}}$

C. $\frac{6}{\sqrt{1-3x^2}}$

D. $\frac{5}{\sqrt{1-4x^2}}$

Answer: A

Solution:

Solution:

Given, $y = \sin^{-1}\left(6x\sqrt{1-9x^2}\right)$

$\Rightarrow y = \sin^{-1}\left(2 \cdot 3x\sqrt{1-(3x)^2}\right)$

Put $3x = \sin\theta \Rightarrow y = \sin^{-1}(2\sin\theta \cdot \cos\theta)$

$\Rightarrow y = \sin^{-1}(\sin 2\theta)$

$\Rightarrow y = 2\theta \Rightarrow y = 2\sin^{-1}(3x)$

$\Rightarrow \frac{dy}{dx} = \frac{2}{\sqrt{1-9x^2}}(3)$

$\Rightarrow \frac{dy}{dx} = \frac{6}{\sqrt{1-9x^2}}$

Question 81

If $y = (\sin x)^x + \sin^{-1}\sqrt{x}$, then $\frac{dy}{dx}$

Options:

A. $(\sin x)^x[\cot x + \log \sin x] + \frac{1}{\sqrt{1-x}} \times \frac{1}{2\sqrt{x}}$

B. $(\sin x)[x \cot x + \log \sin x] + \frac{1}{\sqrt{1-x}} \times \frac{1}{2\sqrt{x}}$

C. $(\sin x)^x[x \cot x + \log \sin x] + \frac{1}{\sqrt{1-x}} \times \frac{1}{2\sqrt{x}}$

D. None of the above

Answer: C

Solution:

Solution:
Given, $y = (\sin x)^x + \sin^{-1}x \ldots$ (i)
Let $u = (\sin x)^x \ldots$ (ii)
Then, Eq. (i) becomes,
$y = u + \sin^{-1}\sqrt{x} \ldots$ (iii)
On taking log both sides of Eq. (ii), we get
$\log u = x \log \sin x$
On differentiating both sides w.r.t., x, we get

$\frac{1}{u}\frac{du}{dx} = x\frac{d}{dx}(\log \sin x) + \log \sin x \frac{d}{dx}(x)$

$\Rightarrow \frac{du}{dx} = u\left[x \times \frac{1}{\sin x}\frac{d}{dx}(\sin x) + \log \sin x(1)\right]$

$\Rightarrow \frac{du}{dx} = (\sin x)^x\left[\frac{x}{\sin x} \times \cos x + \log \sin x\right]$

$\Rightarrow \frac{du}{dx} = (\sin x)^x[x \cot x + \log \sin x] \ldots$ (iv)

On differentiating both sides of Eq. (iii) w.r.t. x, we get

$\frac{dy}{dx} = \frac{du}{dx} + \frac{1}{\sqrt{1-(\sqrt{x})^2}}\frac{d}{dx}(\sqrt{x})$

$\therefore \frac{dy}{dx} = (\sin x)^x[x \cot x + \log \sin x] + \frac{1}{\sqrt{1-x}} \times \frac{1}{2\sqrt{x}}$

[from Eq. (iv)]

Question 82

The side of an equilateral triangle is increasing at the rate of 2cm / s. If the side of the triangle is 20cm^2 the rate of area increasing is

Options:

A. $20\sqrt{3}cm^2$

B. $20cm^2$

C. $60cm^2$

D. $\frac{20\sqrt{3}}{3}cm^2$

Answer: A

Solution:

Solution:

Let a be the side of an equilateral triangle and A be the area of an equilateral triangle. Then, $\frac{da}{dt} = 2cm / s$

We know that, area of an equilateral triangle

$A = \frac{\sqrt{3}}{4}a^2$

On differentiating both sides w.r.t. t, we get

$\frac{dA}{dt} = \frac{\sqrt{3}}{4} \times 2a \times \frac{da}{dt}$

$\Rightarrow \frac{dA}{dt\&} = \frac{\sqrt{3}}{4} \times 2 \times 20 \times 2 \quad [\text{given } a = 20]$

$\therefore \frac{dA}{dt} = 20\sqrt{3}cm^2 / s$

Thus, the rate of area increasing is $20\sqrt{3}cm^2 / s$.

Question 83

The maximum value $Z = 11x + 7y$, subject to $x \le 3$, $y \le 2$, $x \ge 0$ and $y \ge 0$ is

Options:

A. 44

B. 46

C. 54

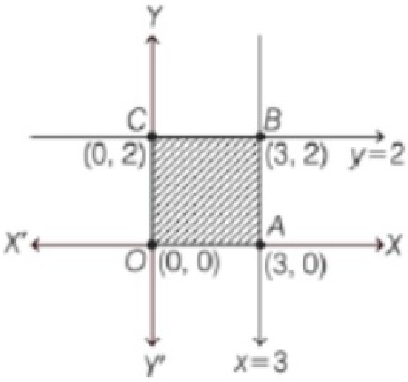
D. 47

Answer: D

Solution:

Solution:

Maximise $Z = 11x + 7y$, subject to the constraints $x \le 3, y \le 2, x \ge 0, y \ge 0$



The shaded region as shown in the figure as OABC is bounded and the coordinates of corner points are (0, 0), (3, 0), (3, 2) and (0, 2) respectively.

Corner points	Corresponding value of Z
(0, 0)	0
(3, 0)	33
(3, 2)	47← Maximum
(0, 2)	14

Hence, Z is maximum at (3, 2) and its maximum value is 47 .

Question 84

The minimum value $Z = 13x - 15y$ subject to $x + y \le 7$, $2x - 3y + 6 \ge 0$, $x \ge 0$ and $y \ge 0$ is

Options:

A. 80

B. −21

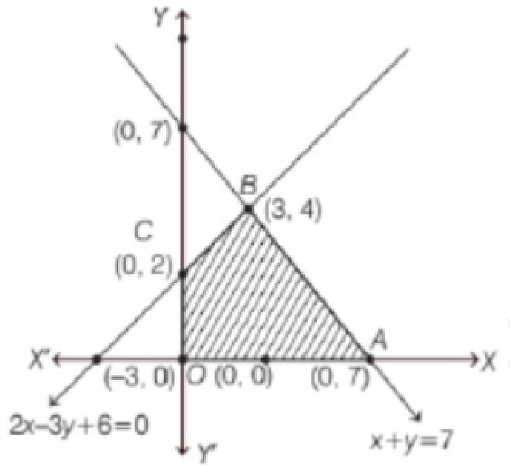
C. −30

D. 91

Answer: C

Solution:

Solution:
Minimize $Z = 13x - 15y$ subject to the constraints $x + y \le 7$, $2x - 3y + 6 \ge 0$, $x \ge 0$, $y \ge 0$



Shaded region shown as OABC is bounded and coordinates of its corner points are (0, 0), (7, 0), (3, 4) and (0, 2) respectively,

Corner points	Corresponding value of Z
(0, 0)	0
(7, 0)	91
(3, 4)	-21
(0, 2)	$-30 \leftarrow$ Minimum

Hence, the minimum value of Z is (-30) at $(0, 2)$.

Question 85

If $z_1, z_2, \ldots, z_n$ are complex numbers such that $z_1| = z_2| = \ldots = z_n| = 1$, then $z_1 + z_2 + \ldots + z_n|$ is equal to

Options:

A. $z_1 z_2 z_3 \ldots z_n|$

B. $z_1| + z_2| + \ldots + z_n|$

C. $\left| \frac{1}{z_1} + \frac{1}{z_2} + \ldots + \frac{1}{z_n} \right|$

D. n

Answer: C

Solution:

Solution:

Given, $|z_1| = |z_2| = \ldots = |z_n| = 1$

$\Rightarrow |z_1|^2 = |z_2|^2 = \ldots = |z_n|^2 = 1$

$\Rightarrow z_1\overline{z_1} = z_2\overline{z_2} = \ldots = z_n\overline{z_n} = 1$

$\Rightarrow \bar{z}_1 = \frac{1}{z_1}, \bar{z}_2 = \frac{1}{z_2}, \ldots, \bar{z}_n = \frac{1}{z_n}$ ... (i)

Now, $|z_1 + z_2 + \ldots + z_n|$

$= |\overline{z_1 + z_2 + \ldots + z_n}| = |\bar{z}_1 + \bar{z}_2 + \ldots + \bar{z}_n|$

$= \left|\frac{1}{z_1} + \frac{1}{z_2} + \ldots + \frac{1}{z_n}\right|$ [using Eq. (i)]

Question 86

If $2\alpha = -1 - i\sqrt{3}$ and $2\beta = -1 + i\sqrt{3}$, then $5\alpha^4 + 5\beta^4 + 7\alpha^{-1}\beta^{-1}$ is equal to

Options:

A. -1

B. 2

C. 0

D. 1

Answer: B

Solution:

Solution:

Given, $2\alpha = -1 - i\sqrt{3}$ and $2\beta = -1 + i\sqrt{3}$

$\therefore\ \alpha + \beta = -1$ and $\alpha\beta = 1$

Now, $5\alpha^4 + 5\beta^4 + \frac{7}{\alpha\beta} = 5[\{(\alpha+\beta)^2 - 2\alpha\alpha\beta\}^2 - 2(\alpha\beta)^2]$

$= 5[\{(-1)^2 - 2 \times 1\}^2 - 2(1)^2] + \frac{7}{1}$

$= 5[(1-2)^2 - 2] + 7 = 2$

Question 87

If a fair coin is tossed 20 times and we get head n times, then probability that n is odd, is

Options:

A. $\frac{1}{2}$

B. $\frac{1}{6}$

C. $\frac{5}{8}$

D. $\frac{7}{8}$

Answer: A

Solution:

Solution:

Probability of getting head in one trial, $p = \frac{1}{2}$ and probability of not getting head,

$q = \frac{1}{2}$

Probability of getting head odd times

$$= {}^{20}C_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^{19} + {}^{20}C_3\left(\frac{1}{3}\right)^3\left(\frac{1}{2}\right)^{17}$$

$$+\ldots + {}^{20}C_{19}\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^{19}$$

$$= \frac{1}{2^{20}}[{}^{20}C_1 + {}^{20}C_3 + \ldots + {}^{20}C_{19}]$$

$$= \frac{1}{2^{20}} \times 2^{20-1} = \frac{2^{19}}{2^{20}} = \frac{1}{2}$$

Question 88

If the records of a hospital show that 10% of the cases of a certain disease are fatal. If 6 patients are suffering from the disease, then the probability that only three will die is

Options:

A. 8748×10^{-5}

B. 1458×10^{-5}

C. 1468×10^{-6}

D. 41×10^{-6}

Answer: B

Solution:

Solution:

Since, the probability of person die, due to suffering from a disease is 10%.

$\therefore\ p = \frac{10}{100} = \frac{1}{10}$ and $q = \frac{9}{10}$

Total number of patients, $n = 6$

$\therefore$ Required probability $= {}^{6}C_3\left(\frac{1}{10}\right)^3\left(\frac{9}{10}\right)^3$

$$= \frac{6\cdot 5\cdot 4}{3\cdot 2\cdot 1} \times \frac{1}{1000} \times \frac{9\times 9\times 9}{1000}$$

$$= \frac{2}{10^5} \times 729 = 1458 \times 10^{-5}$$

Question 89

Area bounded by the curve $x = 0$ and $x + 2|y| = 1$ is

Options:

A. $\frac{1}{4}$

B. $\frac{1}{2}$

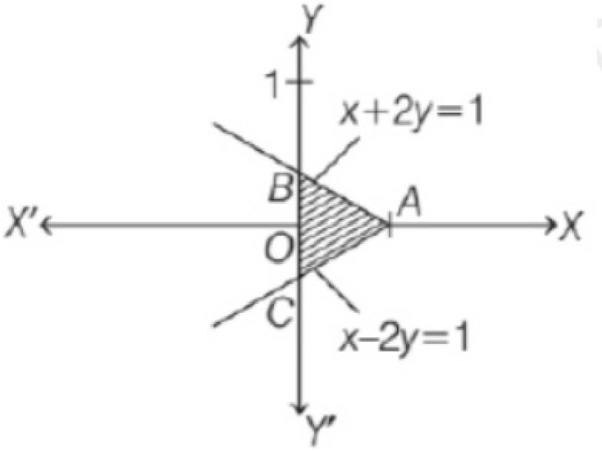
C. 1

D. 2

Answer: B

Solution:

Solution:
Given curves are $x = 0$ and $x + 2|y| = 1$
Now, $x + 2|y| = 0$
When $y > 0$, $x + 2y = 1$; when $y < 0$, $x - 2y = 1$



∴ Area of bounded region ABC

$$= 2AOB = 2\int_0^1 \left(\frac{1-x}{2}\right)dx$$

$$= \left[x - \frac{x^2}{2}\right]_0^1 = \left[1 - \frac{1}{2} - (0-0)\right] = \frac{1}{2}$$

Question 90

The area bounded by the curves $y = \sqrt{5 - x^2}$ and $y = |x - 1|$ is

Options:

A. $\left(\frac{5\pi}{4} - 2\right)$ sq units

B. $\frac{(5\pi - 2)}{4}$ sq units

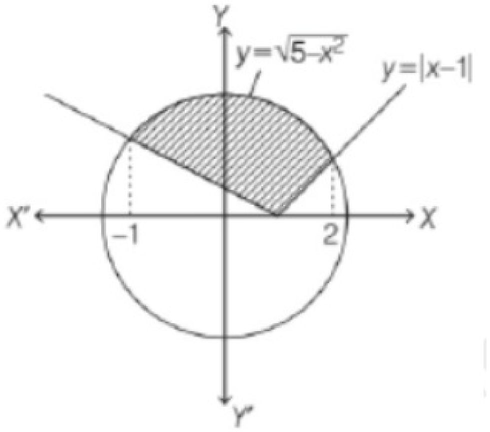
C. $\frac{(5\pi - 2)}{2}$ sq units

D. $\left(\frac{\pi}{2}-5\right)$ sq units

Answer: B

Solution:

Solution:
Given, $y=\sqrt{5-x^2} \Rightarrow y^2+x^2=5$
and $y=|x-1|$



∴ Required area

$$=\int_{-1}^{2}\sqrt{5-x^2}\,dx-\int_{-1}^{1}(1-x)\,dx-\int_{1}^{2}(x-1)\,dx$$

$$=\left[\frac{x}{2}\sqrt{5-x^2}+\frac{5}{2}\sin^{-1}\frac{x}{\sqrt{5}}\right]_{-1}^{2}-\left[x-\frac{x^2}{2}\right]_{-1}^{1}-\left[\frac{x^2}{2}-x\right]_{1}^{2}$$

$$=\left[1+\frac{5}{2}\sin^{-1}\frac{2}{\sqrt{5}}+1+\frac{5}{2}\sin^{-1}\frac{1}{\sqrt{5}}\right]$$

$$-\left[1-\frac{1}{2}-\left(-1-\frac{1}{2}\right)\right]-\left[2-2-\left(\frac{1}{2}-1\right)\right]$$

$$=2+\frac{5}{2}\sin^{-1}\left(\frac{2}{\sqrt{5}}\sqrt{1-\frac{1}{5}}+\frac{1}{\sqrt{5}}\sqrt{1-\frac{4}{5}}\right)-\frac{5}{2}$$

$$=\frac{5}{2}\sin^{-1}(1)-\frac{1}{2}=\frac{5\pi}{4}-\frac{1}{2}=\left(\frac{5\pi-2}{4}\right)\text{ sq units}$$

Question 91

The value of $\int_{3}^{5}\frac{x^2}{x^2-4}dx$ is

Options:

A. $2-\log_e\left(\frac{15}{7}\right)$

B. $2+\log_e\left(\frac{15}{7}\right)$

C. $2+4\log_e 3-4\log_e 7+4\log_e 5$

D. $2-\tan^{-1}\left(\frac{15}{7}\right)$

Answer: B

Solution:

Solution:

$\int_3^5 \frac{x^2}{x^2-4}dx = \int_3^5 \left(\frac{x^2-4}{x^2-4} + \frac{4}{x^2-4}\right)dx$

$= \int_3^5 \left(1 + \frac{4}{x^2-4}\right)dx = \left[x + \frac{4}{2\times 2}\log_e\left(\frac{x-2}{x+2}\right)\right]_3^5$

$= \left[5 + \log_e\left(\frac{5-2}{5+2}\right) - 3 - \log_e\left(\frac{3-2}{3+2}\right)\right]$

$= 2 + \log_e\left(\frac{3}{7}\right) - \log_e\left(\frac{1}{5}\right)$

$= 2 + \log_e\left(\frac{3}{7}\times\frac{5}{1}\right) = 2 + \log_e\left(\frac{15}{7}\right)$

Question 92

The particular solution of the differential equation $\frac{dy}{dx} = \frac{y+1}{x^2-x}$, when x = 2 and y = 1 is

Options:

A. $xy = 3x - 4$

B. $xy = 2x - 2$

C. $xy = 4x - 6$

D. $xy = -x + 4$

Answer: A

Solution:

Solution:

Given, differential equation

$\frac{dy}{dx} = \frac{y+1}{x^2-x} \Rightarrow \frac{dy}{y+1} = \frac{dx}{x^2-x}$

On integrating both sides, we get

$\int \frac{dy}{y+1} = \int \frac{dx}{x^2-x}$

Now, $\frac{1}{x^2-x} = \frac{1}{x(x-1)} = \frac{A}{x} + \frac{B}{x-1}$

$\therefore \frac{1}{x(x-1)} = \frac{A}{x} + \frac{B}{x-1}$ . . . (i)

$\Rightarrow 1 = A(x-1) + B(x)$

Putting $x = 0$, then

$1 = A(0-1) \Rightarrow A = -1$

Putting $x - 1 = 0$, then $x = 1$

$\therefore 1 = A(0) + B(1)$

$\Rightarrow B = 1$

From Eq. (i), we get

$\int \frac{dy}{y+1} = \int -\frac{1dx}{x} + \int \frac{1}{x-1}dx$

$\Rightarrow \log(y+1) = -\log x + \log(x-1) + \log C$

$\Rightarrow \log(y+1) + \log x - \log(x-1) = \log C$

$\Rightarrow \log\left\{\frac{x(y+1)}{x-1}\right\} = \log C$

$\Rightarrow \dfrac{x(y+1)}{x-1} = C \ldots$ (ii)
On putting $x = 2$ and $y = 1$ in Eq. (ii), we get
$\dfrac{2(1+1)}{2-1} = C \Rightarrow C = (2)(2) = 4$
Putting value of $C = 4$ in Eq. (ii), we get
$\dfrac{x(y+1)}{x-1} = 4$
$\Rightarrow xy + x = 4x - 4 \quad \Rightarrow \quad xy = 3x - 4$

Question 93

The differential equation of all circles which passes through the origin and whose centre lies on Y -axis, is

Options:

A. $(x^2 - y^2)\frac{dy}{dx} - 2xy = 0$

B. $(x^2 - y^2)\frac{dy}{dx} + 2xy = 0$

C. $(x^2 - y^2)\frac{dy}{dx} - xy = 0$

D. $(x^2 - y^2)\frac{dy}{dx} + xy = 0$

Answer: A

Solution:

Solution:
Let $x^2 + y^2 - 2ky = 0$
On differentiating w.r.t x, we get
$2x + 2y\frac{dy}{dx} - 2k\frac{dy}{dx} = 0$
$\Rightarrow k = \dfrac{x}{\left(\frac{dy}{dx}\right)} + y$
From Eq. (i),
$x^2 + y^2 - 2\left(\dfrac{x}{(dy/dx)} + y\right)y = 0$
$\Rightarrow (x^2 - y^2)\frac{dy}{dx} - 2xy = 0$

Question 94

The solution of the equation $(x^2 + xy)dy = (x^2 + y^2)dx$ is

Options:

A. $\log x = \log(x - y) + \frac{y}{x} + C$

B. $\log x = 2\log(x - y) + \frac{y}{x} + C$

C. $\log x = \log(x - y) + \frac{y}{x} + C$

D. None of the above

Answer: B

Solution:

Solution:

Given, $(x^2 + xy)dy = (x^2 + y^2)dx$

$\Rightarrow \frac{dy}{dx} = \frac{x^2 + y^2}{x^2 + xy}$

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\Rightarrow v + x\frac{dv}{dx} = \frac{x^2 + v^2x^2}{x^2 + x^2v}$

$\Rightarrow v + x\frac{dv}{dx} = \frac{1 + v^2}{1 + v}$

$\Rightarrow \frac{xdv}{dx} = \frac{1 + v^2}{1 + v} - v$

$\Rightarrow = \frac{1 + v^2 - v - v^2}{1 + v} = \frac{1 - v}{1 + v}$

$\Rightarrow dv\left(\frac{1 + v}{1 - v}\right) = \frac{dx}{x} \Rightarrow dv\left(-1 + \frac{2}{1 - v}\right) = \frac{dx}{x}$

On integrating both sides, we get

$-v - 2\log(1 - v) = \log x + C$

$\Rightarrow \frac{y}{x} - 2\log\left(1 - \frac{y}{x}\right) = \log x + C$

$\Rightarrow \frac{y}{x} - 2\log(x - y) + 2\log(x - y) + C = \log x + C$

Question 95

The value of $\int_{\pi/6}^{\pi/2}\left(\frac{1 + \sin 2x + \cos 2x}{\sin x \cos x}\right)dx$ is equal to

Options:

A. 16

B. 8

C. 4

D. 1

Answer: D

Solution:

Solution:

$\int_{\pi/6}^{\pi/2}\left(\frac{1 + \sin 2x + \cos 2x}{\sin x \cos x}\right)dx$

$$= \int_{\pi/6}^{\pi/2} \left(\frac{1 + \sin x \cos x + 2\cos^2 x - 1}{(\sin x + \cos x)} \right) dx$$
$$= \int_{\pi/6}^{\pi/2} \left(\frac{2\cos x(\sin x + \cos x)}{(\sin x + \cos x)} \right) dx$$
$$= \int_{\pi/6}^{\pi/2} 2\cos x \, dx = 2[\sin x]_{\pi/6}^{\pi/2}$$
$$= 2\left(\sin\frac{\pi}{2} - \sin\frac{\pi}{6}\right) = 2\left(1 - \frac{1}{2}\right) = 2 \times \frac{1}{2} = 1$$

Question 96

How many 5-digit telephone number can be constructed using the digits 0 to 9 , if each number starts with 67 and no digits appears more than once?

Options:

A. 335

B. 336

C. 338

D. 337

Answer: B

Solution:

Solution:
Since, telephone number start with 67 , so two digits is already fixed. Now, we have to do arrangement of three digits from remaining eight digits.
∴ Possible number of ways $= {}^8P_3$
$$= \frac{8!}{(8-3)!} = \frac{8!}{5!} = 8 \times 7 \times 6 = 336 \text{ ways}$$

Question 97

The number of selecting atleast 4 candidates from 8 candidates is

Options:

A. 270

B. 70

C. 163

D. None of these

Answer: C

Solution:

Solution:
Required number of selections
$= {}^8C_4 + {}^8C_5 + {}^8C_6 + {}^8C_7 + {}^8C_8$
$= 70 + 56 + 28 + 8 + 1 = 163$

Question 98

If $f(x) = \frac{2x-1}{x+5}$, $x \neq -5$, then $f^{-1}(x)$ is equal to

Options:

A. $\frac{x+5}{2x-1}$, $x \neq \frac{1}{2}$

B. $\frac{5x+1}{2-x}$, $x \neq 2$

C. $\frac{x-3}{2x+1}$, $x \neq \frac{1}{2}$

D. $\frac{5x-1}{2-x}$, $x \neq 2$

Answer: D

Solution:

Solution:
Let $y = \frac{2x-1}{x+5}$
$\Rightarrow x = \frac{5y+1}{2-y}$
$\therefore f^{-1}(x) = \frac{5x-1}{2-x}$, $x \neq 2$

Question 99

If $f(x) = \frac{\alpha x}{x+1}$, $x \neq -1$, for what values of α is $f(f(x)) = x$?

Options:

A. $\sqrt{2}$

B. $-\sqrt{2}$

C. -1

D. 2

Answer: C

Solution:

Solution:

$f(x) = \frac{\alpha x}{x+1}, x \neq 1$

$\therefore\ f(f(x)) = f\left(\frac{\alpha x}{x+1}\right) = \frac{\alpha\left(\frac{\alpha x}{x+1}\right)}{\left(\frac{\alpha x}{x+1}\right)+1} = \frac{\alpha^2 x}{\alpha x + x + 1}$

$\Rightarrow\ \frac{\alpha^2 x}{\alpha x + x + 1} = x$ [given]

$\Rightarrow\ x[\alpha^2 - \alpha x - x - 1] = 0$

$\Rightarrow\ x(\alpha+1)(\alpha - 1 - x) = 0$

$\Rightarrow\ x = 0$ or $\alpha + 1 = 0$

or $\alpha = 1 + x$ $[\because \alpha - 1 - x \neq 0]$

$\Rightarrow\ \alpha = -1$ or $\alpha = 1 + x$

$\therefore\ \alpha = -1$

[$\because \alpha = 1 + x$ gives value for particular x, not for all x]

Question 100

If A and B are two events with $P(A^c) = 0.3$, $P(B) = 0.4$ and $P(A \cap B^c) = 0.5$. Then, $P[(B) / (A \cap B^c)]$ is equal to

Options:

A. $\frac{1}{4}$

B. $\frac{1}{3}$

C. $\frac{1}{2}$

D. $\frac{2}{3}$

Answer: A

Solution:

Solution:

Given, $P(A^c) = 0.3$, $P(B) = 0.4$

and $P(A \cap B^c) = 0.5$

$\because\ P(A^c) = 0.3$

$\Rightarrow\ P(A) = 1 - P(A^c) = 0.7$

and $P(B) = 0.4 \Rightarrow P(B^C) = 1 - P(B) = 0.6$

Consider, $P(A \cap B^C) = P(A) - P(A \cap B)$

$0.5 = 0.7 - P(A \cap B)$

$\Rightarrow\ P(A \cap B) = 0.2$

Now, $P\left[\frac{B}{(A \cup B^C)}\right] = \frac{P[B \cap (A \cup B^C)]}{P(A \cup B^C)}$

$= \frac{P[(B \cap A) \cup (A \cap B^C)]}{P(A) + P(B^C) - P(A \cap B^c)}$

$$= \frac{P(B \cap A)}{0.7 + 0.6 - 0.5} = \frac{0.2}{0.8} = \frac{1}{4}$$
